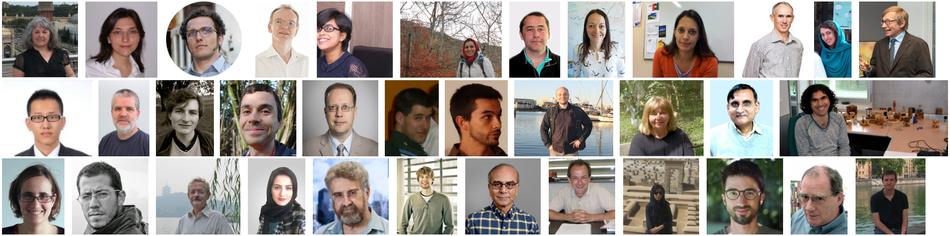
Graph identification problems

selected topics

Florent Foucaud

GTA workshop, IPM Isfahan, January 2021




PARIS
LYON
BORDEAUX
MARSEILLE


PARIS
CLERMONT-FERRAND
LYON
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NORD-PAS-DE-CALAIS
PICARDIE
HAUTE-NORMANDIE
BASSE-NORMANDIE
BRETAGNE
PAYS DE LA LOIRE
CENTRE
CHAMPAGNE-ARDENNE
LORRAINE
ALSACE
BOURGOGNE
FRANCHE-COMTE
LIMOUSIN
AUVERGNE
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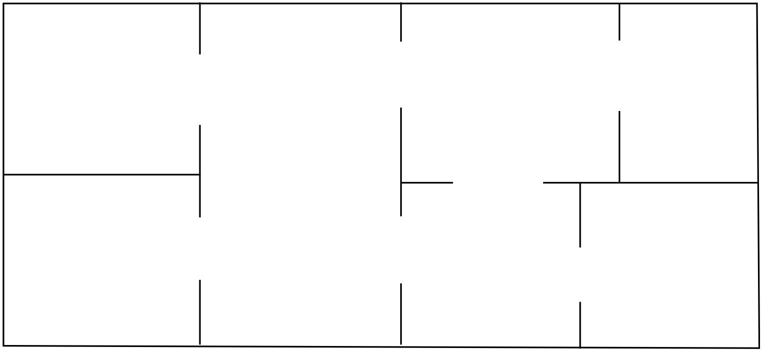
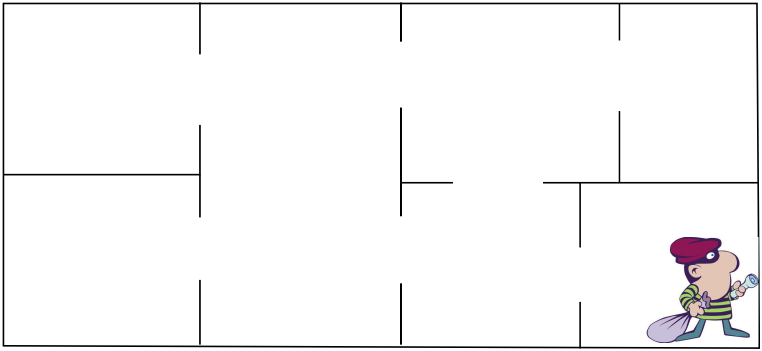
LANGUEDOC-ROUSSILLON
PROVENCE-ALPES-COTE D'AZUR
CORSE


MICHELIN


Saint Nectaire



Locating a burglar



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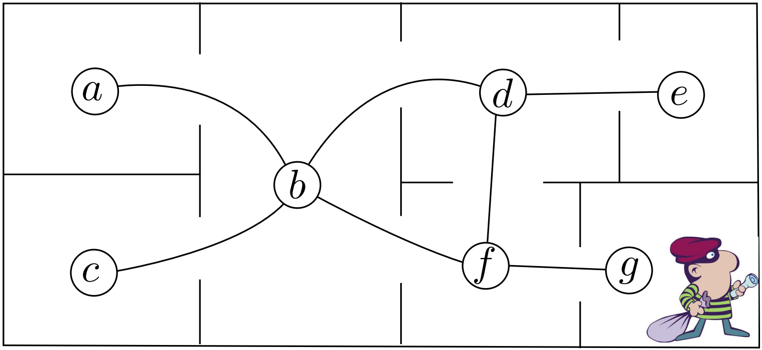
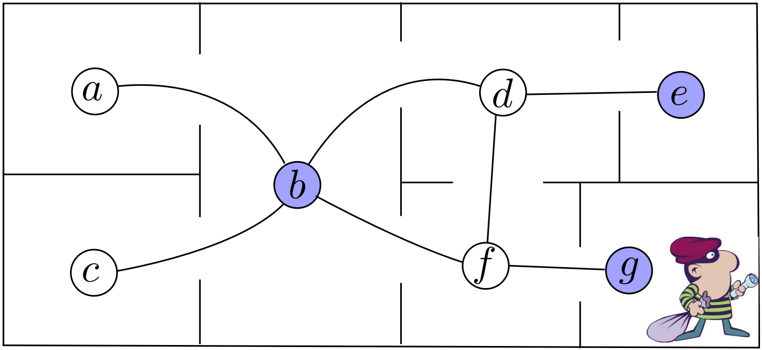
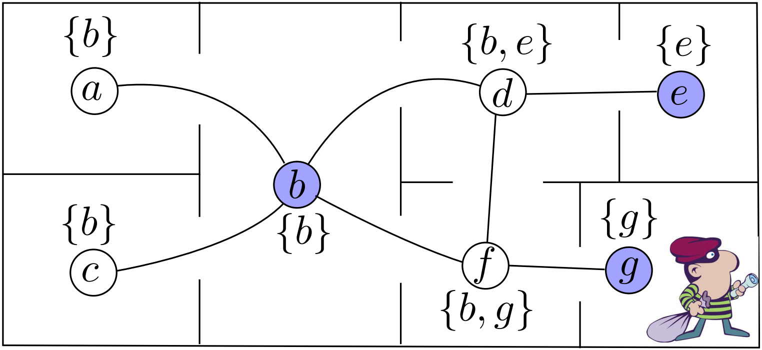




Detectors can detect movement in their room and adjacent rooms



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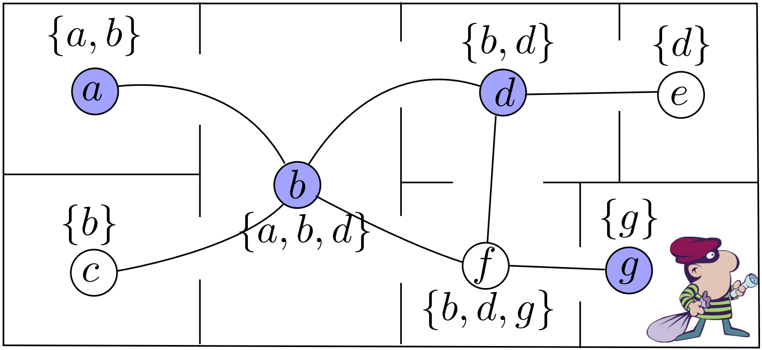

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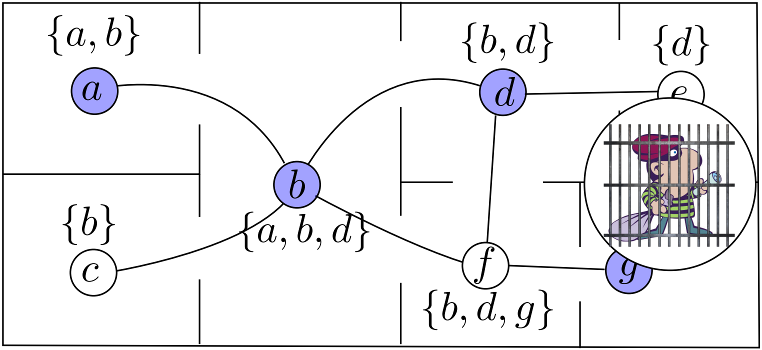

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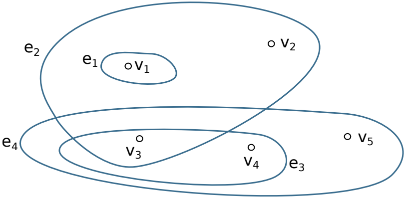
Separating sets in hypergraphs

Definition - Separating set (Rényi, 1961 )

Hypergraph $(X, \mathscr{E})$. A separating set is a subset $C \subseteq X$ such that each edge $e \in \mathscr{E}$ contains a distinct subset of C.



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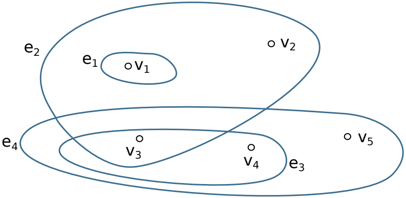

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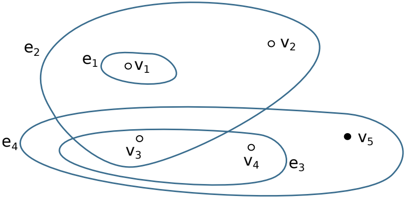

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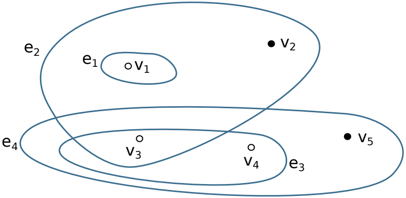

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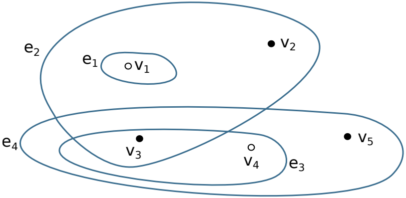

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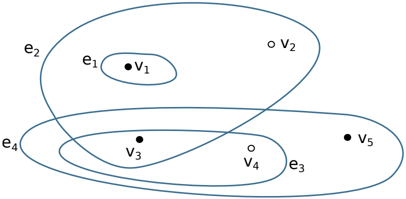

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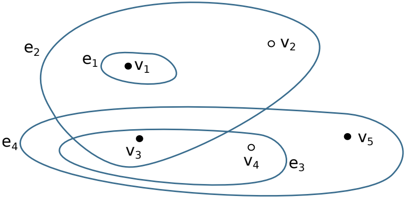

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Also known as Separating system, Distinguishing set, Test cover, Distinguishing transversal, Discriminating code...

Applications

- network-monitoring, fault detection (burglar)
- medical diagnostics: testing samples for diseases (*test cover*)
- biological identification (attributes of individuals)
- learning theory: teaching dimension
- machine learning: V-C dimension (Vapnik, Červonenkis, 1971)
- graph isomorphism: canonical representation of graphs (Babai, 1982)
- logic definability of graphs (Kim, Pikhurko, Spencer, Verbitsky, 2005)

Proposition

For a hypergraph $(X,\mathscr{E})$, a separating set C has size at least $\log_2(|\mathscr{E}|)$.

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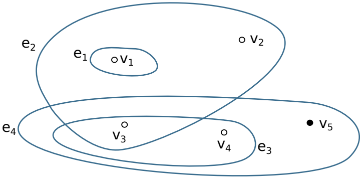
Which are the "problematic" vertices?



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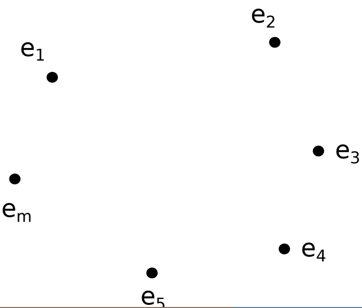

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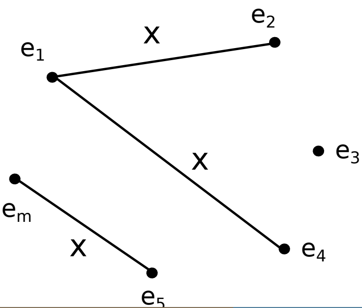

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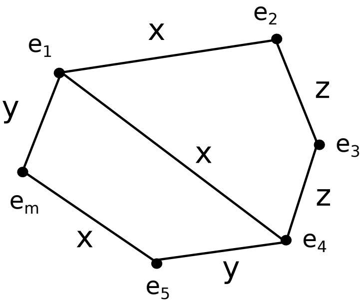

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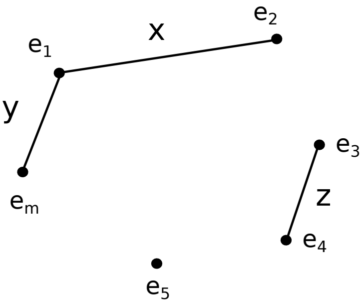

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This destroys all cycles in G!

General bounds, Bondy's theorem

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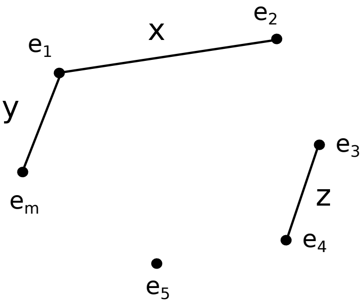

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So, there are at most $|\mathscr{E}|-1$ "problematic" vertices. $\rightarrow$ Find one "non-problematic vertex" and omit it. □

Some example problems

Special graph-based cases of separating sets in hypergraphs:

- identifying codes
- **open neighbourhood locating-dominating sets**
- path/cycle identifying covers
- separating path systems

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A variation:

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Geometric versions: e.g. seperating points using disks in Euclidean space

Distance-based identification:

- **resolving sets** (metric dimension)
- centroidal locating sets
- tracking paths problem

Open neighbourhood location-domination in graphs

G: undirected graph $N(u)$: set of neighbours of v

Definition - OLD set (Seo, Slater, 2010  )

Subset D of $V(G)$ such that:

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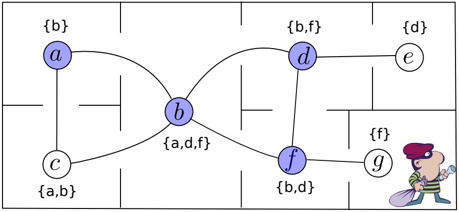
Notation. $OLD(G)$: OLD number of G,
minimum size of an OLD-set in G



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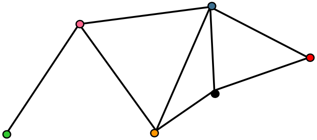

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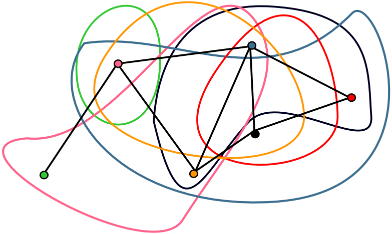

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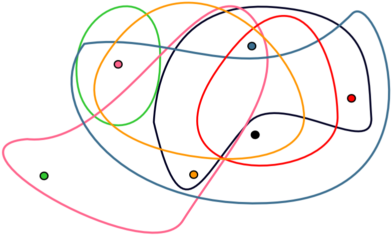

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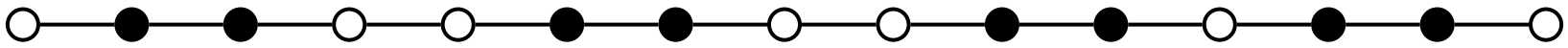

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OLD-number: $OLD(P_n) \approx \lceil \frac{2n}{3} \rceil$

Remark

Not all graphs have an OLD set!

An **isolated vertex** cannot be totally dominated.

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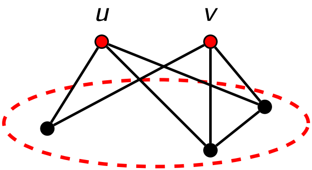

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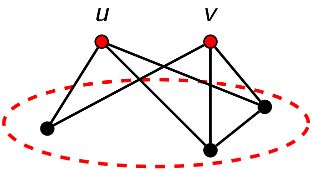

Proposition

A graph is **locatable** if and only if it has no **isolated vertices** and **open twins**.

Lower bound on $OLD(G)$

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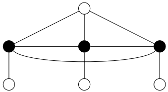
- D is a **separating set**: $\forall u \neq v$ of $V(G)$, $N(u) \cap D \neq N(v) \cap D$

Proposition

G locatable graph on n vertices: $\lceil \log_2(n+1) \rceil \leq OLD(G)$. (Tight.)

Proof: For any OLD-set D, we must assign to each vertex, a distinct non-empty subset of D: $n \leq 2^{|D|} - 1$.

$OLD(G) = \log_2(n+1)$



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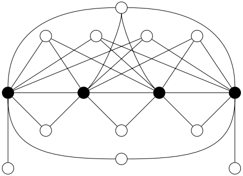
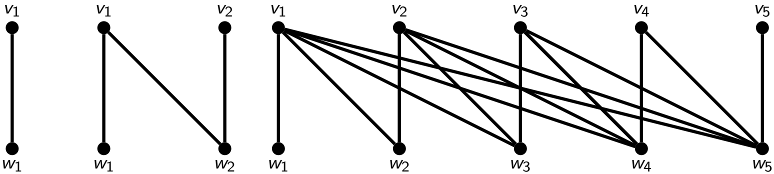


□

Definition - Half-graph H_k (Erdős, Hajnal, 1983  )

Bipartite graph on vertex sets $\{v_1, \ldots, v_k\}$ and $\{w_1, \ldots, w_k\}$, with an edge $\{v_i, w_j\}$ if and only if $i \leq j$.



$H_1 = P_2$ $\quad$ $H_2 = P_4$ $\quad$ H_5

Upper bound on $OLD(G)$?

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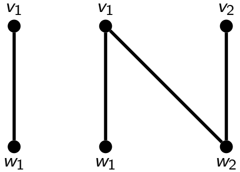

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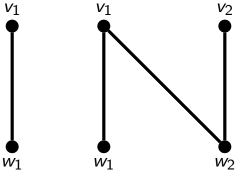

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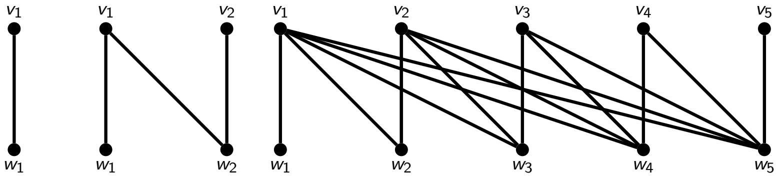

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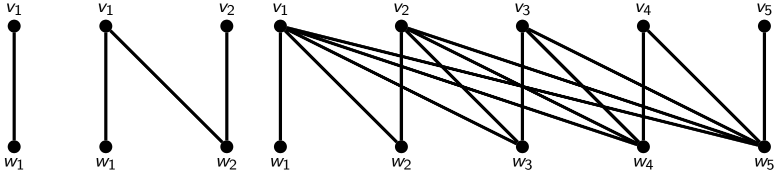

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Some vertices are **forced** to be in any OLD-set because of **domination** or **location**

Proposition

For every half-graph H_k of order $n = 2k$, $OLD(H_k) = n$.

Characterizing "bad graphs" for OLD-sets

Theorem (F., Ghareghani, Roshany Tabrizi, Sharifani, 2020+   )

Let G be a connected locatable graph of order n.

Then, $OLD(G) = n$ if and only if G is a half-graph.

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By induction, G' is a half-graph. We can conclude that G is a half-graph too.

□

Location-domination in graphs

Definition - Locating-dominating set (Slater, 1980's) 

$D \subseteq V(G)$ locating-dominating set of G:

- for every $u \in V$, $N[v] \cap D \neq \emptyset$ (domination).
- $\forall u \neq v$ of $V(G) \setminus D$, $N(u) \cap D \neq N(v) \cap D$ (location).

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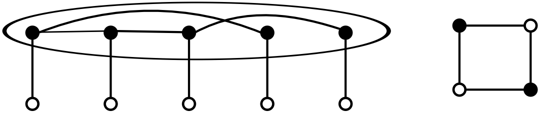
Notation. location-domination number $LD(G)$,
smallest size of a locating-dominating set of G

Domination number: $DOM(P_n) = \lceil \frac{n}{3} \rceil$

Location-domination number: $LD(P_n) = \lceil \frac{2n}{5} \rceil$

Theorem (Domination bound, Ore, 1960's )

G graph of order n, no isolated vertices. Then $DOM(G) \leq \frac{n}{2}$.

Tight examples: 

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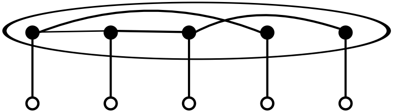

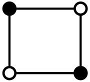

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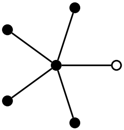

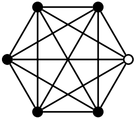

Upper bounds

Theorem (Domination bound, Ore, 1960's )

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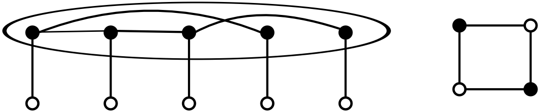
Tight examples:



Theorem (Location-domination bound, Slater, 1980's )

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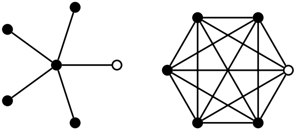

Remark: tight examples contain many twin-vertices!!

Theorem (Domination bound, Ore, 1960's )

G graph of order n, no isolated vertices. Then $DOM(G) \leq \frac{n}{2}$.

Theorem (Location-domination bound, Slater, 1980's )

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G graph of order n, no isolated vertices, no twins. Then $LD(G) \leq \frac{n}{2}$.

Upper bound: a conjecture

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Remark:

- twins are easy to detect
- twins have a trivial behaviour w.r.t. location-domination

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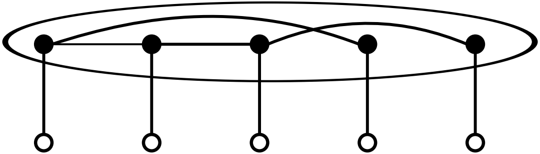
If true, tight: 1. domination-extremal graphs



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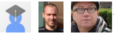
Conjecture (Garijo, González & Márquez, 2014 )

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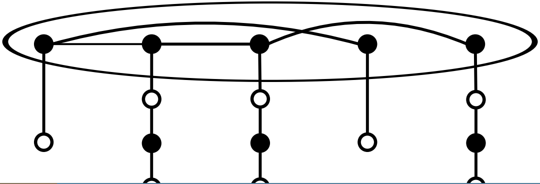
If true, tight: 2. a similar construction



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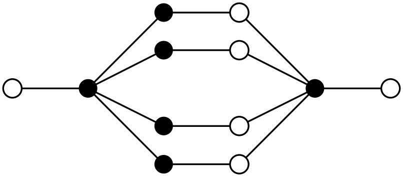
If true, tight: 3. a family with domination number 2



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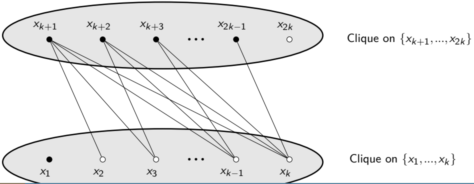
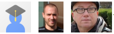
If true, tight: 4. family with dom. number 2: complements of half-graphs



Conjecture (Garijo, González & Márquez, 2014 )

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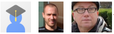
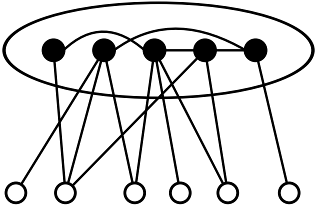
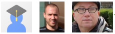
Theorem (Garijo, González & Márquez, 2014 )

Conjecture true if G has independence number $\geq n/2$.
(in particular, if bipartite)

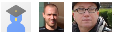
Proof: every vertex cover is a locating-dominating set



Conjecture (Garijo, González & Márquez, 2014 )

G graph of order n, no isolated vertices, no twins. Then $LD(G) \leq \frac{n}{2}$.

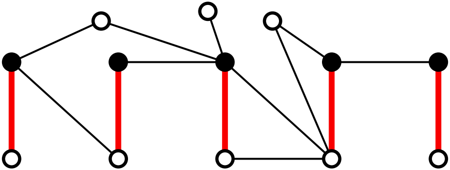
$\alpha'(G)$: matching number of G

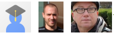
Theorem (Garijo, González & Márquez, 2014 )

If G has no 4-cycles, then $LD(G) \leq \alpha'(G) \leq \frac{n}{2}$.

Proof:

- Consider special maximum matching M
- Select one vertex in each edge of M



Conjecture (Garijo, González & Márquez, 2014 )

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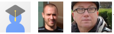
Theorem (Garijo, González & Márquez, 2014 )

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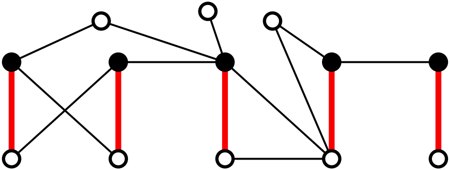

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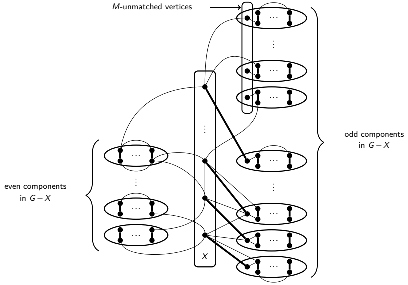
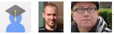
G graph of order n, no isolated vertices, no twins. Then $LD(G) \leq \frac{n}{2}$.

Theorem (F., Henning, 2016 )

Conjecture true if G is cubic.

Proof: Involved argument using maximum matching and Tutte-Berge theorem.



Conjecture (Garijo, González & Márquez, 2014 )

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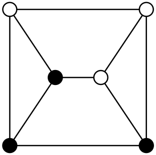
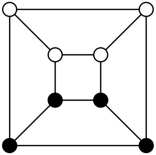
Bound is tight:





Question

Do we have $LD(G) = \frac{n}{2}$ for other cubic graphs?

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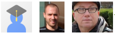
Question

Are there twin-free (cubic) graphs with $LD(G) > \alpha'(G)$?

(if not, conjecture is true)

Conjecture (Garijo, González & Márquez, 2014 )

G graph of order n, no isolated vertices, no twins. Then $LD(G) \leq \frac{n}{2}$.

Theorem (F., Henning, Löwenstein, Sasse, 2016 )

Conjecture true if G is split graph or complement of bipartite graph.

Line graph of G: intersection graph of the edges of G.

Theorem (F., Henning, 2017 )

Conjecture true if G is a line graph.

Proof: By induction on the order, using edge-locating-dominating sets

Conjecture (Garijo, González & Márquez, 2014   )

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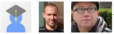
Conjecture (Garijo, González & Márquez, 2014 )

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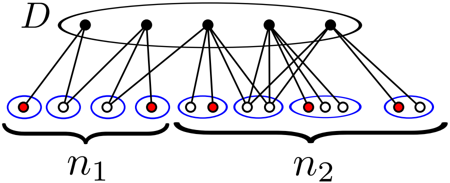
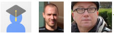


Conjecture (Garijo, González & Márquez, 2014 )

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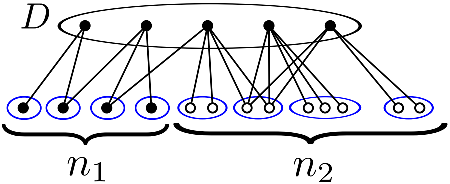
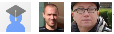
• there is a LD-set of size $|D| + n_1$;



Conjecture (Garijo, González & Márquez, 2014 )

G graph of order n, no isolated vertices, no twins. Then $LD(G) \leq \frac{n}{2}$.

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Proof: • There exists a dominating set D such that each vertex has a private neighbour. We have $|D| \leq n_1 + n_2$.

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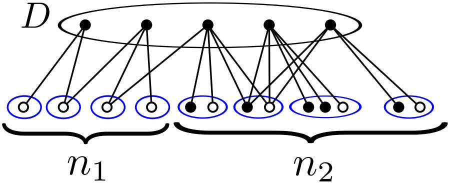
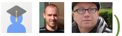


Conjecture (Garijo, González & Márquez, 2014 )

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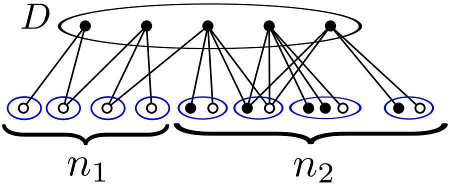

Lower bounds

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Proposition

G identifiable graph on n vertices: $\lceil \log_2(n+1) \rceil \leq OLD(G) \leq LD(G)$.

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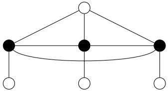
Tight examples:

$OLD(G) = \log_2(n+1)$



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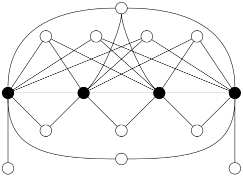

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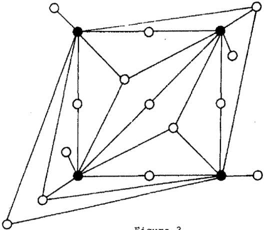
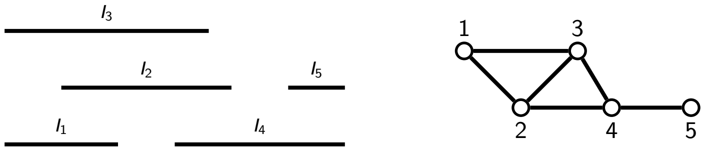


Figure 3.

Definition - Interval graph

Intersection graph of intervals of the real line.



Theorem (F., Mertzios, Naserasr, Parreau, Valicov, 2017    )

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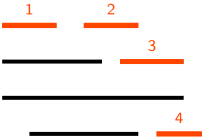

- Identifying code D of size k.
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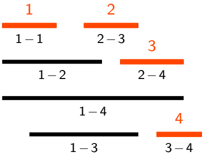

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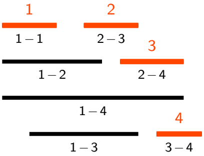

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$\rightarrow n \leq \sum_{i=1}^{k}(k-i) = \frac{k(k+1)}{2}$.

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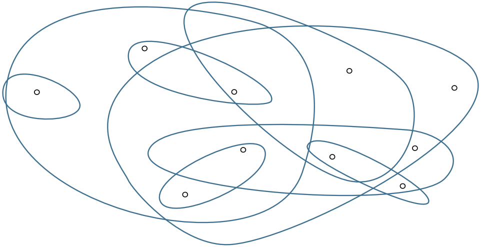
Tight:

Measure of intersection complexity of sets in a hypergraph $(X, \mathscr{E})$
(initial motivation: machine learning, 1971)

A set $S \subseteq X$ is shattered:

for every subset $S' \subseteq S$, there is an edge e with $e \cap S = S'$.



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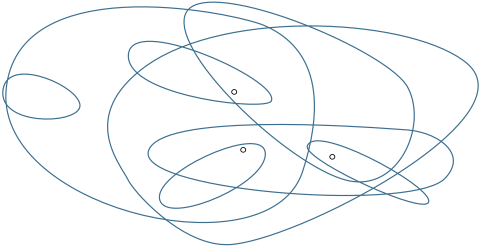

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Vapnik-Červonenkis dimension

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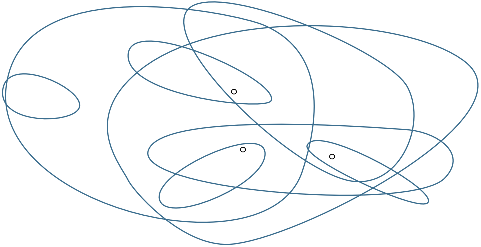

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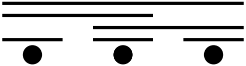
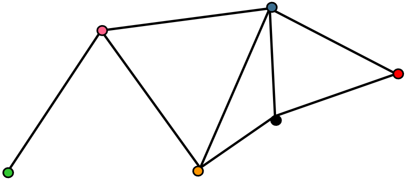
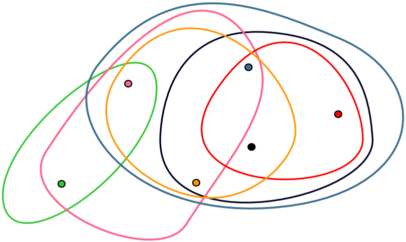


V-C dimension of a graph: V-C dimension of its open/closed neighbourhood hypergraph





Vapnik-Červonenkis dimension - graphs

V-C dimension of a graph: V-C dimension of its open/closed neighbourhood hypergraph

Typically bounded for geometric intersection graphs:

→ interval graphs ($d = 2$), C_4-free graphs ($d = 2$), line graphs ($d = 4$), permutation graphs ($d = 3$), unit disk graphs ($d = 3$), planar graphs ($d = 4$)...

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Theorem (Sauer-Shelah Lemma  )

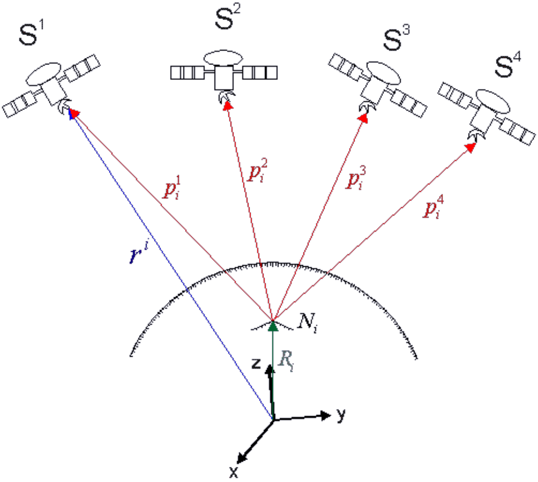
Let H be a hypergraph of V-C dimension at most d. Then, any set S of vertices has at most $|S|^d$ distinct traces.

Corollary

G graph of order n, $LD(G) = k$, V-C dimension $\leq d$. Then $n = O(k^d)$.

Metric dimension

GPS/GLONASS/Galileo/Beidou/IRNSS:
need to know the exact position of 4 satellites + distance to them



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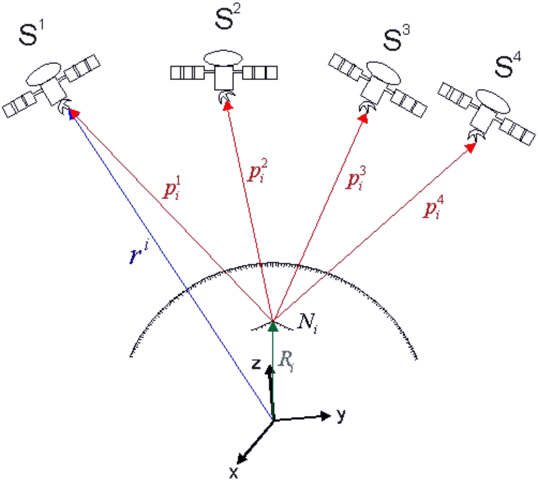

Question

Does the "GPS" approach also work in undirected unweighted graphs?

Now, $w \in V(G)$ distinguishes $\{u, v\}$ if $dist(w, u) \neq dist(w, v)$

Definition - Resolving set (Slater, 1975 - Harary & Melter, 1976)   

$R \subseteq V(G)$ resolving set of G:

$$\forall u \neq v \text{ in } V(G), \text{ there exists } w \in R \text{ that distinguishes } \{u, v\}.$$

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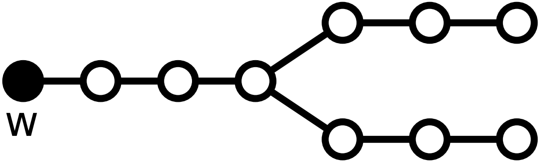

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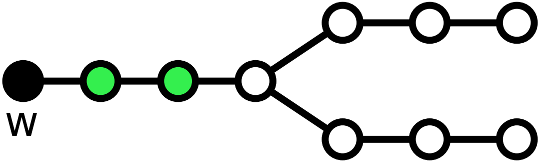

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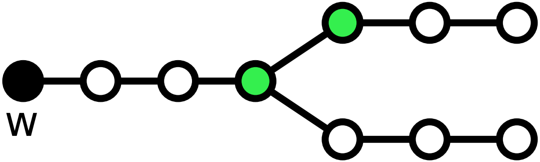

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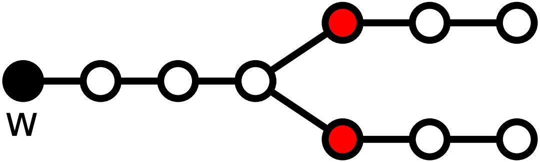

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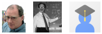
Definition - Resolving set (Slater, 1975 - Harary & Melter, 1976) 

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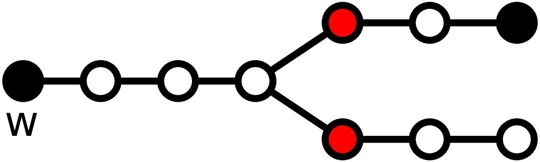

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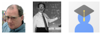
Definition - Resolving set (Slater, 1975 - Harary & Melter, 1976) 

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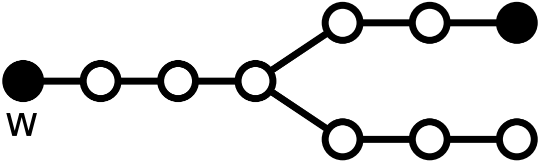

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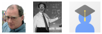
Definition - Resolving set (Slater, 1975 - Harary & Melter, 1976) 

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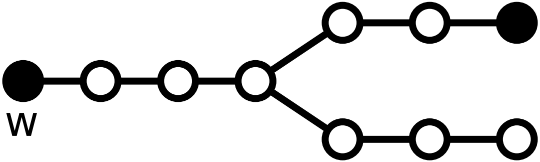

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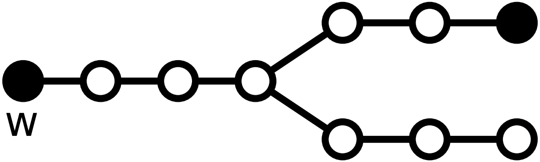

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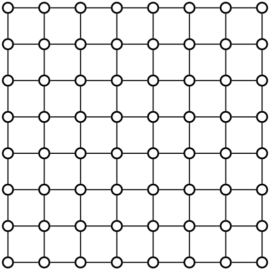

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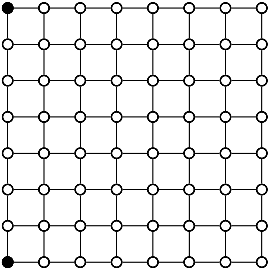
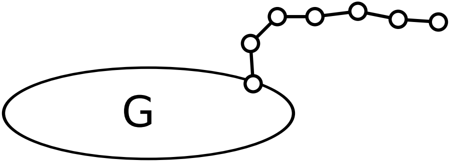


Proposition

For any square grid G, $MD(G) = 2$.

Leg: path with all inner-vertices of degree 2, endpoints of degree ≥ 3 and 1.



Trees

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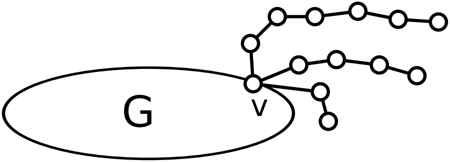

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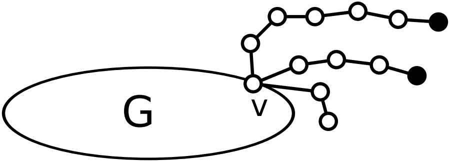

Observation

R resolving set. If v has k legs, at least $k-1$ legs contain a vertex of R.

Simple leg rule: if v has $k \geq 2$ legs, select $k-1$ leg endpoints.

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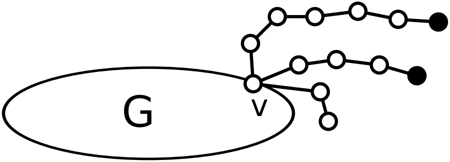

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Theorem (Slater, 1975 )

For any tree, the simple leg rule produces an optimal resolving set.

Bounds with diameter

Example of path: no bound $n \leq f(MD(G))$ possible.

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Theorem (Khuller, Raghavachari & Rosenfeld, 2002   )

G of order n, diameter D, $MD(G) = k$. Then $n \leq D^k + k$.

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G interval graph of order n, $MD(G) = k$, diameter D. Then $n = O(Dk^2)$ i.e. $k = \Omega\left(\sqrt{\frac{n}{D}}\right)$. (Tight.)

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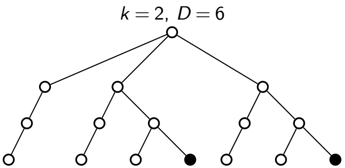
$\rightarrow$ Proofs are similar as for identifying codes.

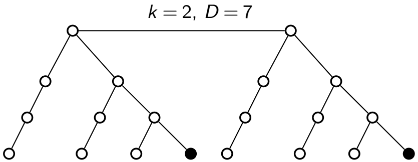
Theorem (Beaudou, Dankelmann, F., Henning, Mary, Parreau, 2018 )

T a tree with diameter D and $MD(T) = k$, then

$$n \leq \begin{cases} \frac{1}{8}(kD+4)(D+2) & \text{if } D \text{ even,} \\ \frac{1}{8}(kD-k+8)(D+1) & \text{if } D \text{ odd.} \end{cases} = \Theta(kD^2)$$

Bounds are tight.





Planar graphs

Using the concept of distance-VC-dimension:

Theorem (Beaudou, Dankelmann, F., Henning, Mary, Parreau, 2018 )

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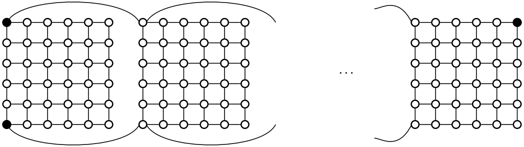
Using the concept of distance-VC-dimension:

Theorem (Beaudou, Dankelmann, F., Henning, Mary, Parreau, 2018 )

G planar with diameter D and $MD(G) = k$, then $n = O(k^4 D^4)$.

Tight? Example with $k = 3$ and $n = \Theta(D^3)$:



Conclusion

Some open problems:

- Conjecture: $LD(G) \leq n/2$ in the absence of twins
- Find tight bounds for id. problems in interesting graph classes

  (beyond e.g. planar graphs)
- Find tight bounds for Metric Dimension in planar graphs of diameter D

  (and other classes)

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THANKS FOR YOUR ATTENTION