Monitoring the edges of a graph using distances

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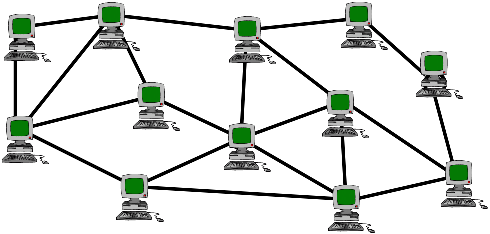
Motivation: Detect failures in a network



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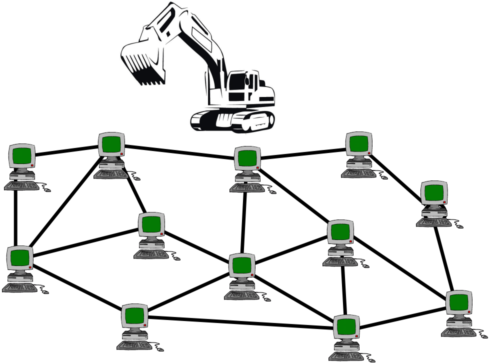

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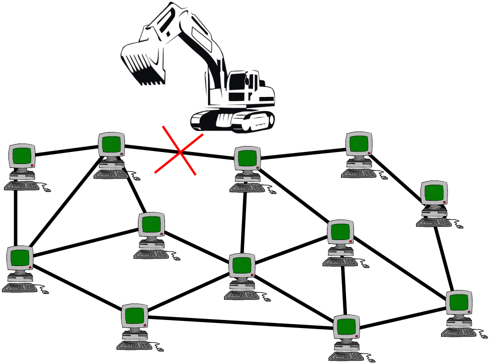

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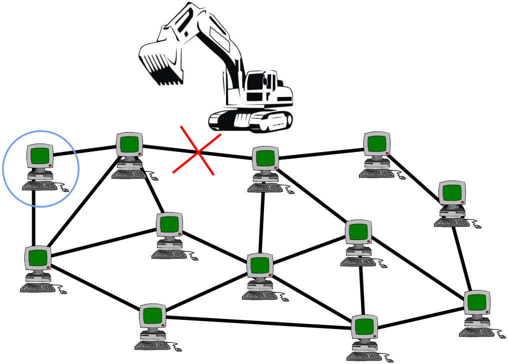

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Monitoring

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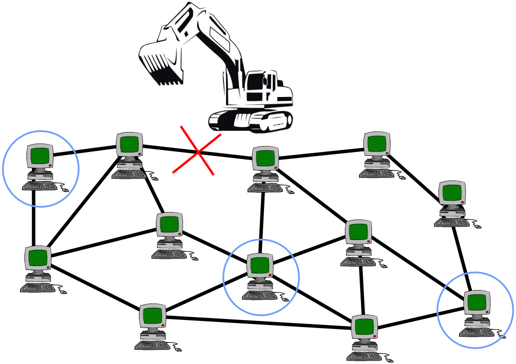


Place distance-detecting probes on the network: a probe p monitors the edges that lie on **all** shortest paths to some vertex x

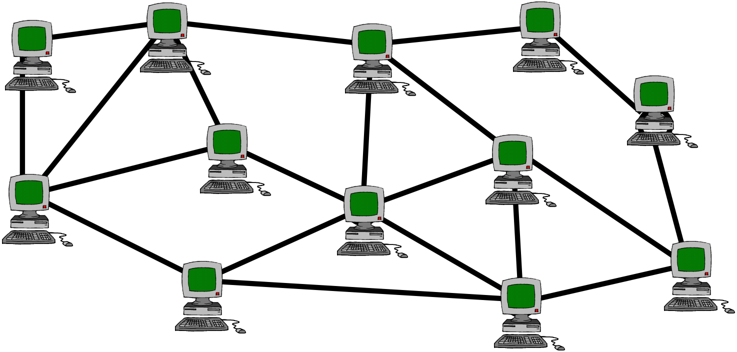
Question

How many probes are enough?

A probe at vertex x monitors the edges that lie on **all** shortest paths to some vertex y

Definition

A set S of vertices of a graph G is distance-edge-monitoring if every edge is monitored by a vertex in S.



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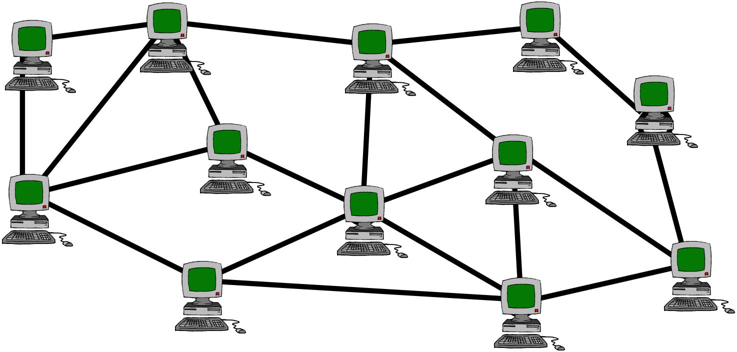
$dem(G)$: smallest size of such a set.



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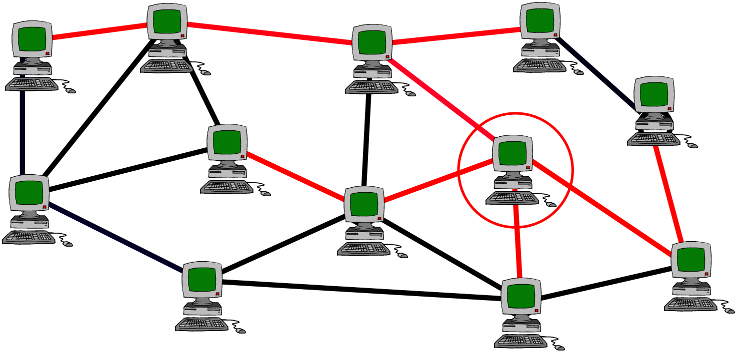

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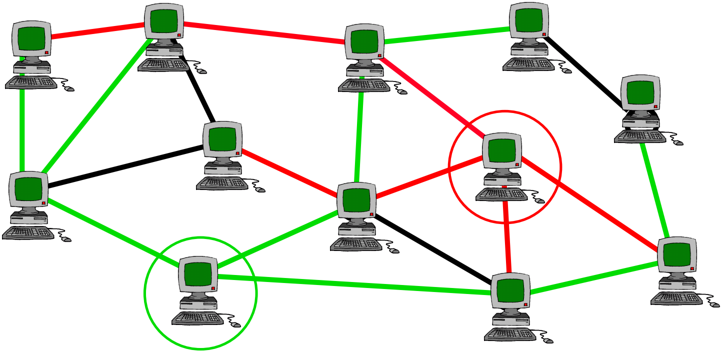

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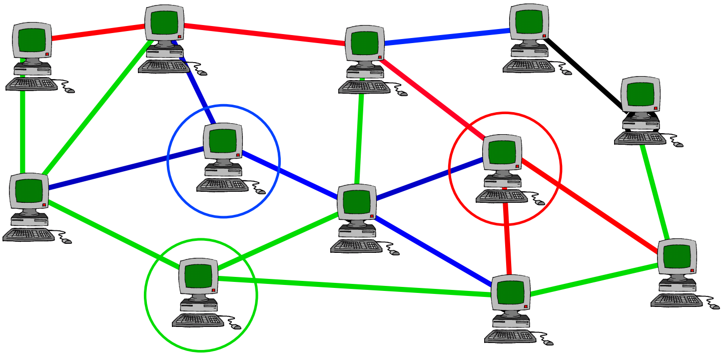

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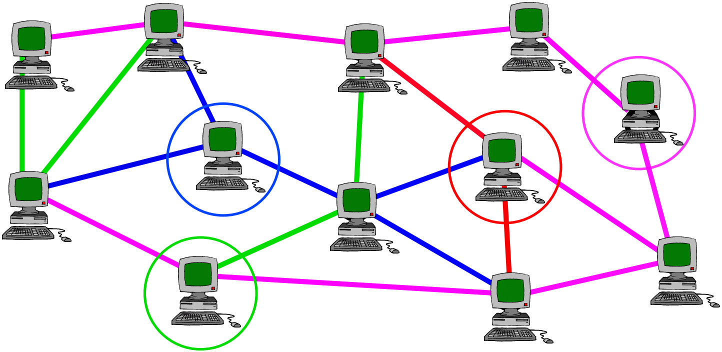

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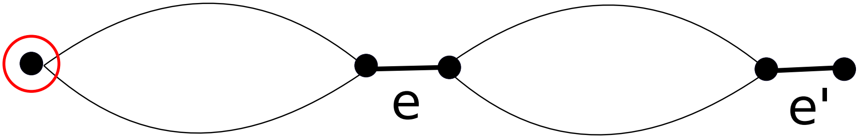
Let S be a distance-edge-monitoring set, and $P(S, e)$ the set of pairs (x, y) s.t. e lies on all shortest paths from $x \in S$ to y.

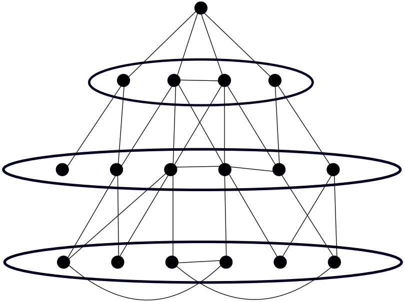
Proposition

For two distinct edges e, e', we have $P(S, e) \neq P(S, e')$.



Structure of edges monitored by a specific vertex

Which are the edges of G monitored by a probe? Do a BFS at given vertex:



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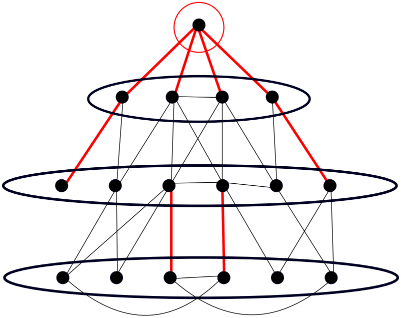

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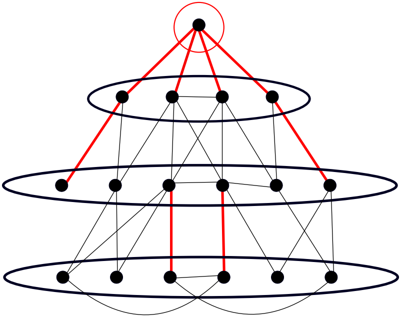

Remarks:

• This set induces a forest. $\Rightarrow$ $dem(G)$ is at least the arboricity of G.
(arboricity: smallest number of forests into which $E(G)$ can be partitioned)

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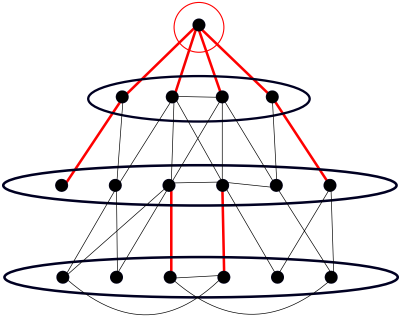

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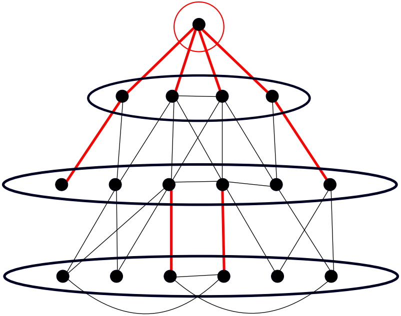

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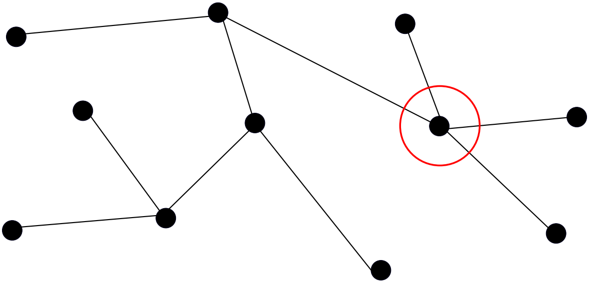
• Every bridge of G is monitored by any vertex

Proposition

For any tree T, $dem(T) = 1$.



Trees

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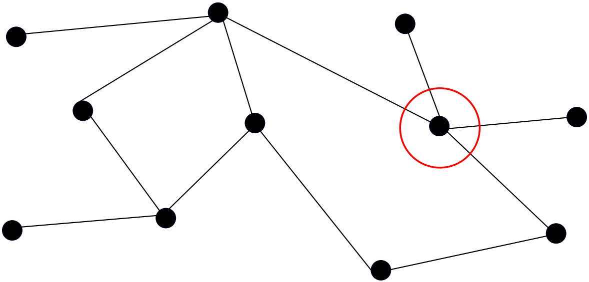
G has $dem(G) = 1$ if and only if G is a tree.



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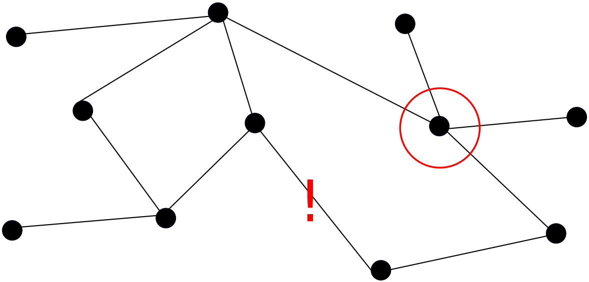

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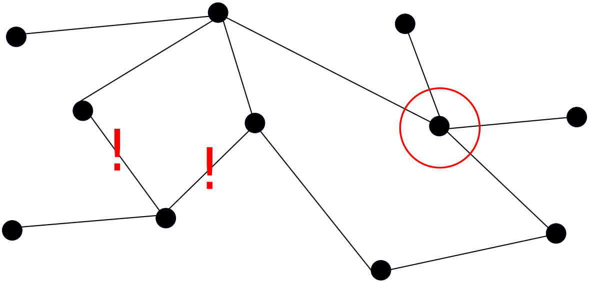

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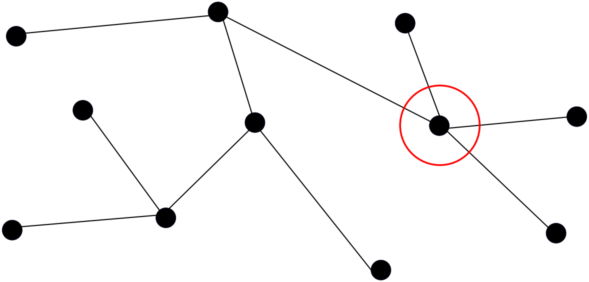
If G is uniyclic, then $dem(G) = 2$.



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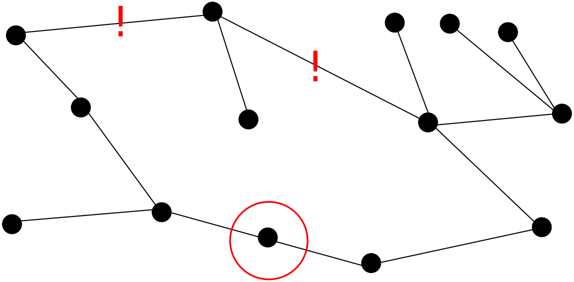

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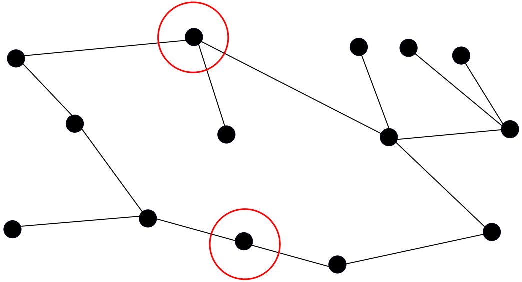
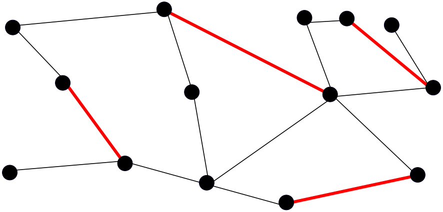


Connection to feedback edge sets

Feedback edge set of G: set S of edges such that every cycle of G has an edge in S

$fes(G)$: smallest size of a feedback edge set of G.



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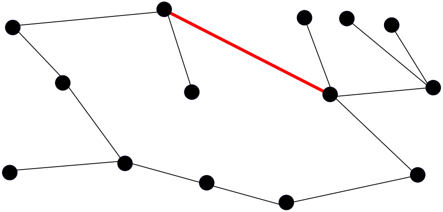

Tree T: $fes(T) = 0$; Unicyclic graph G: $fes(G) = 1$

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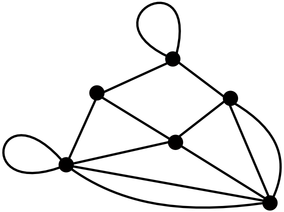
Lemma (Folklore)

If $fes(G) = k$, then G is obtained from a multigraph H of order at most $2k - 2$ and size $3k - 3$ by iteratively subdividing edges and adding degree 1 vertices.



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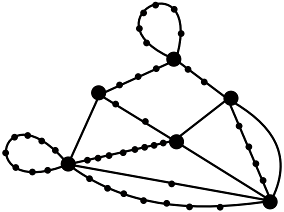

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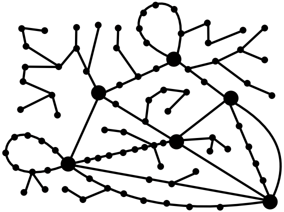

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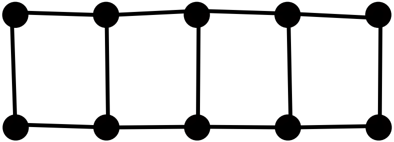
(Tight for a ladder $P_2 \square P_{k+1}$.)



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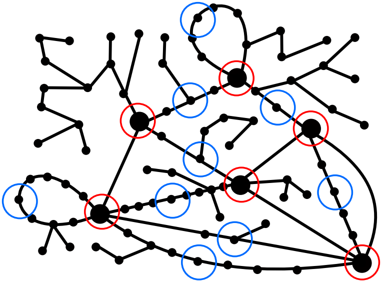
Theorem

For any graph G, we have $dem(G) \leq 5fes(G) - 5$.



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Theorem

For any graph G, we have $dem(G) \leq 2fes(G) - 2$.

DEM
Input: Graph G
Task: Find smallest distance-edge-monitoring set of G

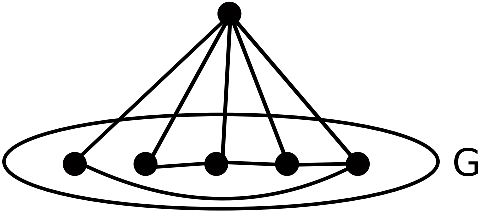
Theorem

DEM is NP-complete.

Proof: reduction from VERTEX COVER:

Lemma

For any graph G of radius at least 4, $dem(G) \times K_1 = vc(G)$.



Theorem

DEM is approximable within a factor of $\ln |E(G)| + 1$ for any graph G.

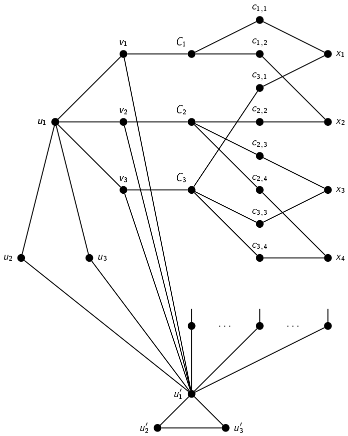
Proof: reduction to SET COVER.

Sets are vertices of G, elements are edges of G.

Theorem

For every $\epsilon > 0$, DEM is NOT approximable within a factor of $(1-\epsilon)\ln|E(G)|$ in polynomial time, unless $P = NP$ (even on subcubic bipartite graphs).
Moreover, the probem is W[2]-hard for parameter solution size.

Proof: reduction from SET COVER.



Open questions

- Conjecture: $dem(G) \leq fes(G) + 1$ (true for $fes(G) = 0, 1, 2$)
- Is DEM NP-hard for planar graphs? Interval graphs?
- Are there approximation/FPT algorithms for nice classes?

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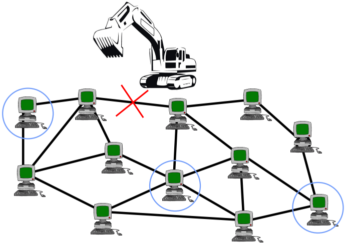

Thanks!