Identifying codes and metric dimension on selected graph classes

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joint work with:
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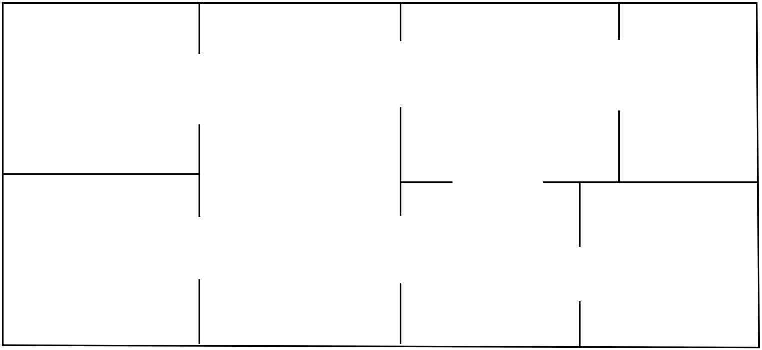
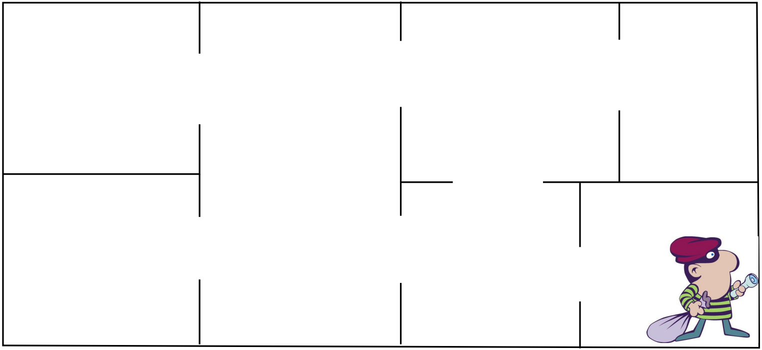
Reza Naserasr (U. Paris-Sud), Petru Valicov (ENS Lyon)

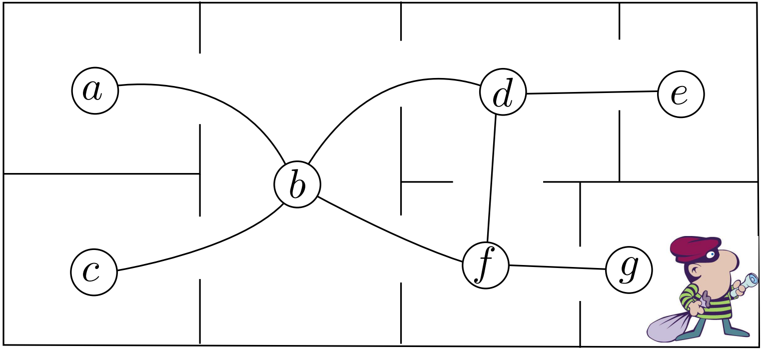
LAMSADE, July 2014

Part I: identifying codes



Locating a burglar in a museum

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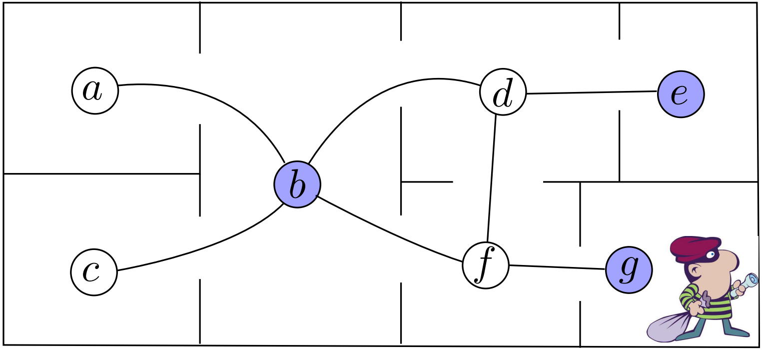
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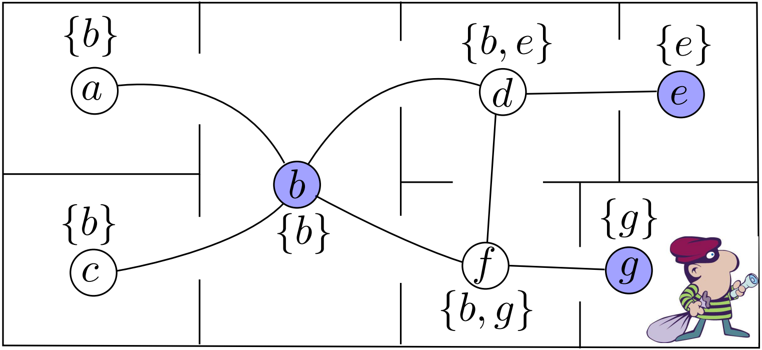
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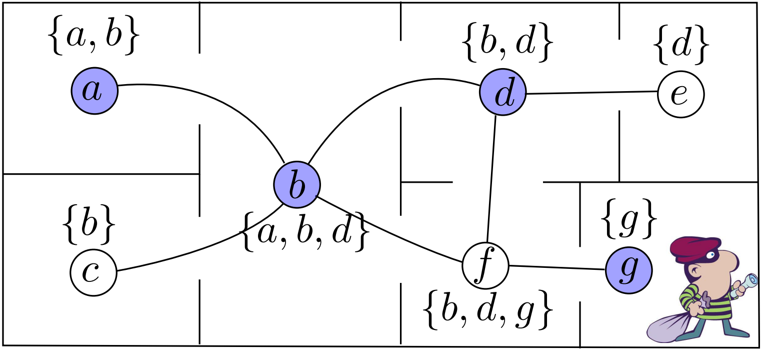
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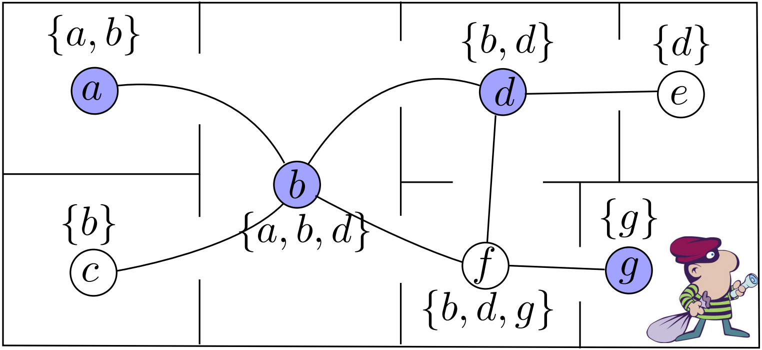
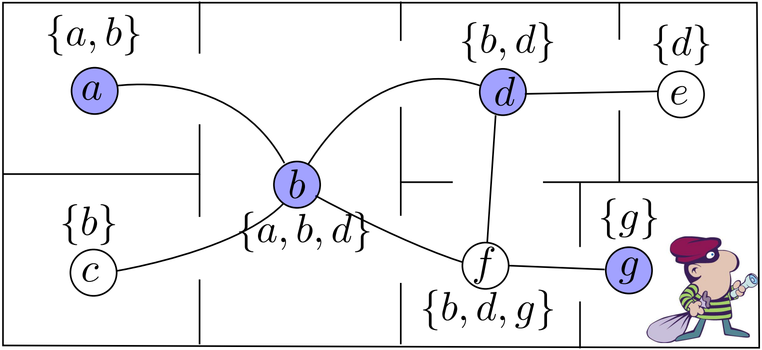


$N[v] = N(v) \cup \{v\}$

$C \subseteq V(G)$ is an identifying code of G (Karpovsky, Chakrabarty, Levitin, 1998):

- for every $u \in V$, $N[v] \cap C \neq \emptyset$ (domination).
- $\forall u \neq v$ of V, $N[u] \cap C \neq N[v] \cap C$ (separation).



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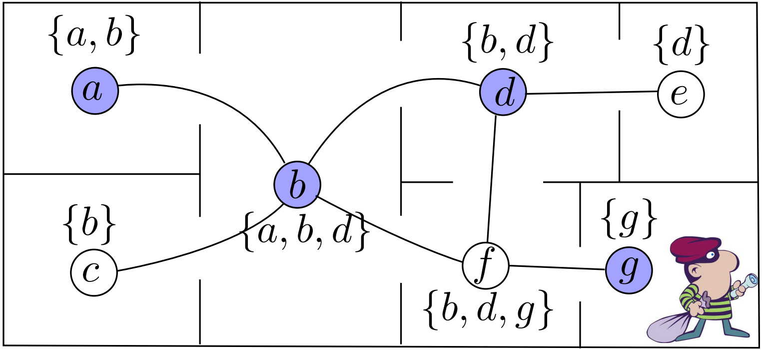
$\gamma^{\text{ID}}(G)$: identifying code number , minimum size of an identifying code of G.



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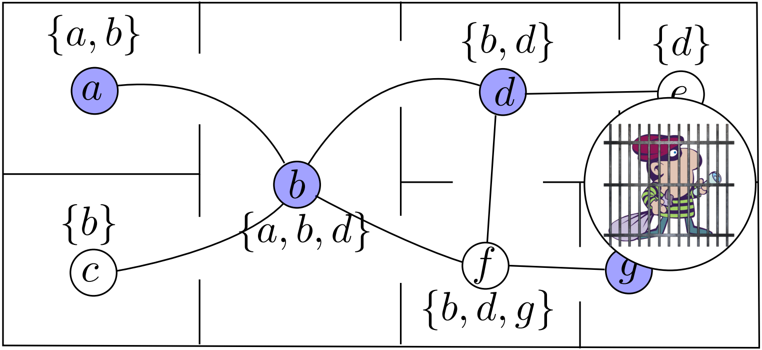

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Theorem (Karpovsky, Chakrabarty, Levitin, 1998 + Gravier & Moncel 2007)

Let G be a nonempty graph on n vertices, then

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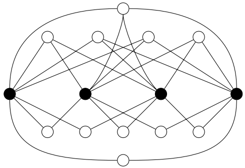

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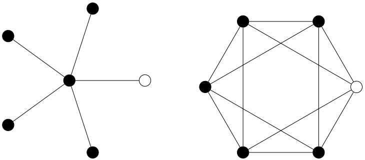

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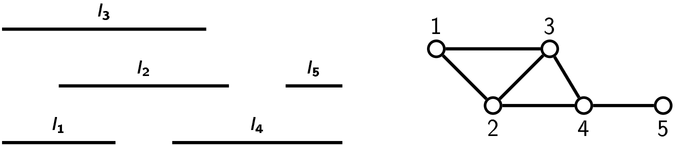
Definition - Interval graph

Intersection graph of intervals of the real line.



Bound for interval graphs

Theorem (F., Mertzios, Naserasr, Parreau, Valicov, 2014+)

If G is an interval graph on n vertices, then $\gamma^{\text{ID}}(G) > \sqrt{2n}$.

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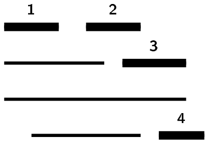

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- Define zones using the right points of code intervals.

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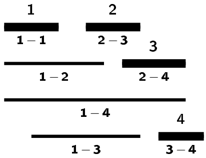

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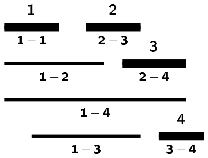

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$\rightarrow n \leq \sum_{i=1}^{k}(k-i) < \frac{k^2}{2}$

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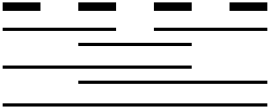
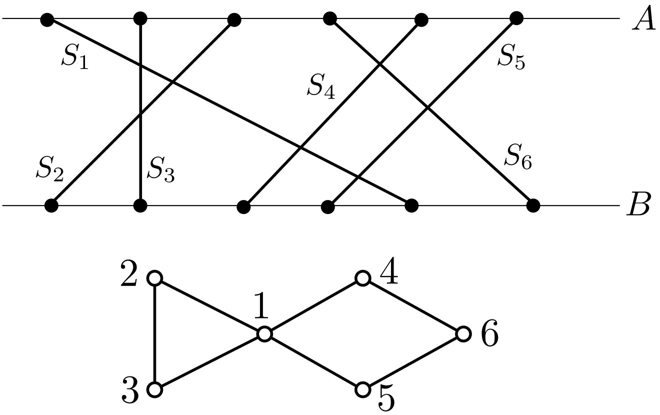


Definition - Permutation graph

Given two parallel lines A and B:
intersection graph of segments joining A and B.



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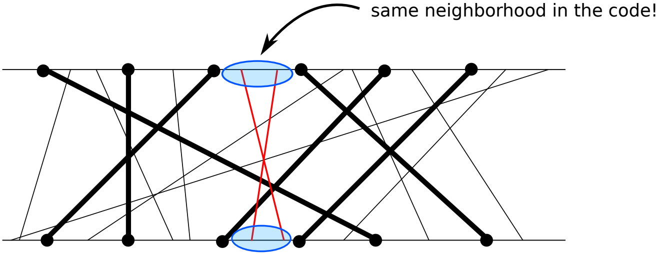

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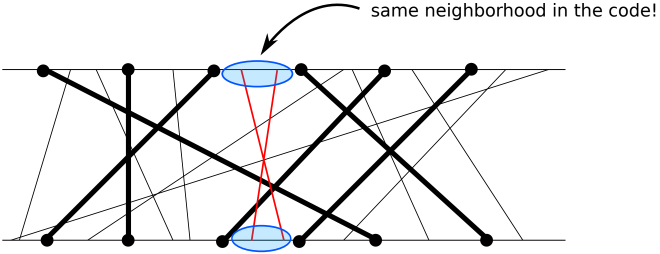

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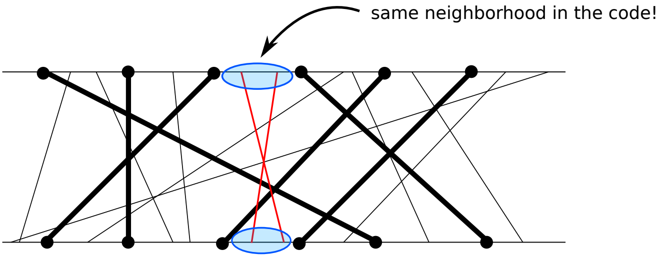

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- Careful counting for the precise bound

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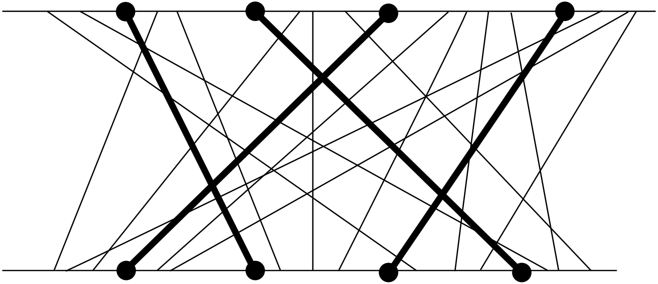

Bounds for subclasses of interval/permutation

Theorem (F., Mertzios, Naserasr, Parreau, Valicov, 2014+)

Let G be a graph on n vertices.

- If G is *unit* interval, then $\gamma^{\text{ID}}(G) \geq \frac{n+1}{2}$.
- If G is *bipartite* permutation, then $\gamma^{\text{ID}}(G) \geq \frac{n-2}{3}$.
- If G is a cograph, then $\gamma^{\text{ID}}(G) \geq \frac{n+2}{2}$.

Vapnis-Chervonenkis dimension

Set $X \subseteq V(G)$ is shattered:
for every subset $S \subseteq X$, there is a vertex v with $N[v] \cap X = S$

V-C dimension of G: maximum size of a shattered set in G

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Theorem (Bousquet, Lagoutte, Li, Parreau, Thomassé, Trunck, 2014+)

Let G be a graph with n vertices, V-C dimension $\leq c$. Then $\gamma^{\text{ID}}(G) \geq n^{1/c}$.

$\rightarrow$ interval graphs ($c = 2$), line graphs ($c = 4$), permutation graphs ($c = 3$), unit disk graphs ($c = 3$), planar graphs ($c = 4$)...

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But better bounds exist:

- planar graphs: $\gamma^{\text{ID}}(G) \geq \frac{n}{7}$ (Slater & Rall, 1984)
- line graphs: $\gamma^{\text{ID}}(G) \geq \frac{3\sqrt{2n}}{4}$ (F., Gravier, Naserasr, Parreau, Valicov, 2013)

IDENTIFYING CODE

INPUT: Graph G, integer k.
QUESTION: Is there an identifying code of G of size k?

- polynomial for graphs of bounded clique-width via MSOL (Courcelle)
- NP-complete for:
  - bipartite (Charon, Hudry, Lobstein, 2003)
  - planar bipartite unit disk (Müller & Sereni, 2009)
  - planar arbitrary girth (Auger, 2010)
  - planar bipartite subcubic (F. 2013)
  - co-bipartite, split (F. 2013)
  - line (F., Gravier, Naserasr, Parreau, Valicov, 2013)

IDENTIFYING CODE

INPUT: Graph G, integer k.
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- $O(\log \Delta)$-approximable (SET COVER)
- constant c-approximation for:
  - planar, $c = 7$ (Slater, Rall, 1984)
  - line, $c = 4$ (F., Gravier, Naserasr, Parreau, Valicov, 2013)
  - interval, $c = 2$ (Bousquet, Lagoutte, Parreau, Thomassé, Trunck, 2014+)
  - unit interval, PTAS
- hard to approximate within $o(\log n)$ for:
  - general graphs (Laifenfeld, Trachtenberg + Suomela 2007)
  - bipartite, split, co-bipartite (F. 2013)
- APX-hard for:
  - line (F., Gravier, Naserasr, Parreau, Valicov, 2013)
  - subcubic bipartite (F. 2013)

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- Trivially FPT for parameter k because $n \leq 2^k \rightarrow$ whole graph is kernel.
- Trivial *polynomial* kernel for interval, permutation, line, planar...

Theorem (F., Mertzios, Naserasr, Parreau, Valicov, 2014+)

IDENTIFYING CODE is NP-complete for graphs that are both interval and permutation.

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Reduction from 3-DIMENSIONAL MATCHING:

- INPUT: A, B, C sets and $\mathscr{T} \subset A \times B \times C$ triples
- QUESTION: is there a perfect 3-dimensional matching $M \subset T$, i.e., each element of $A \cup B \cup C$ appears exactly once in M?

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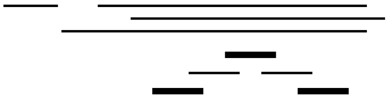
Main idea: an interval can separate pairs of intervals **far away** from each other (without affecting what lies in between)

Complexity - gadgets

Dominating gadget: ensure all intervals are dominated and most, separated.



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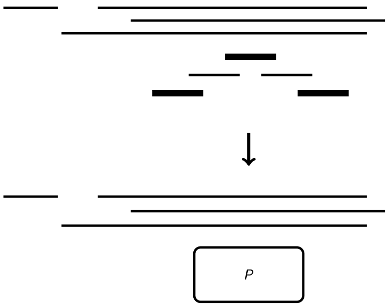
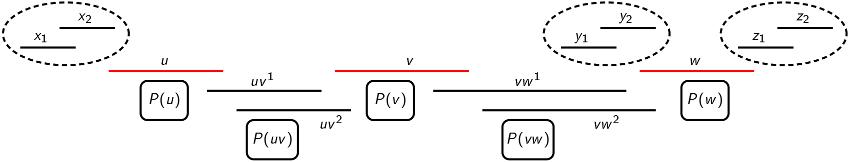


Transmitter gadget: to separate $\{uv^1, uv^2\}$ and $\{vw^1, vw^2\}$, either:

1. take only v into solution, or
2. take both u, w — and separate pairs $\{x_1, x_2\}$, $\{y_1, y_2\}$, $\{z_1, z_2\}$ "for free".



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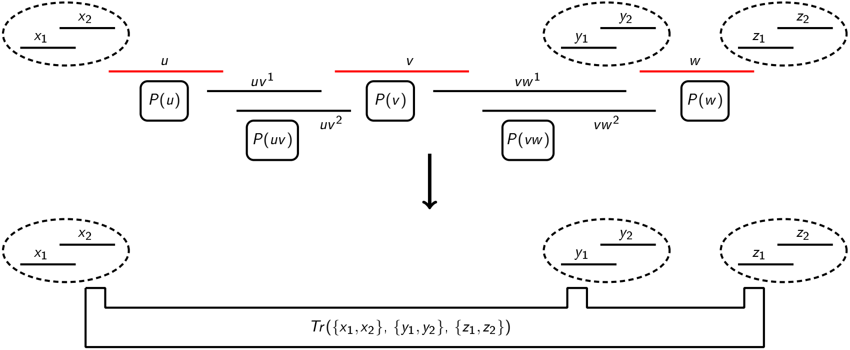
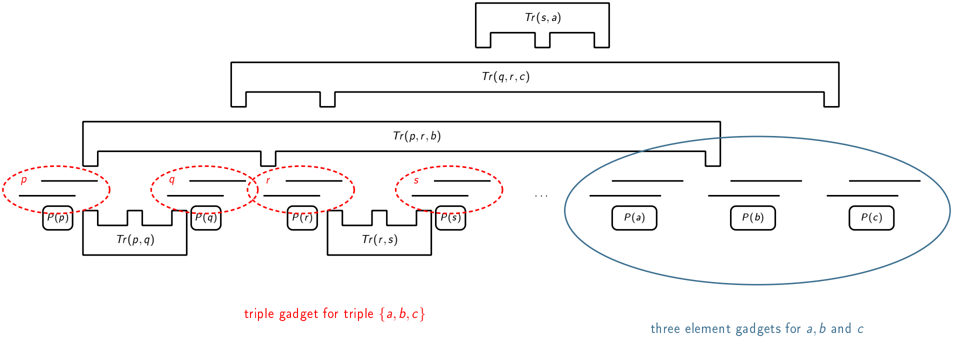
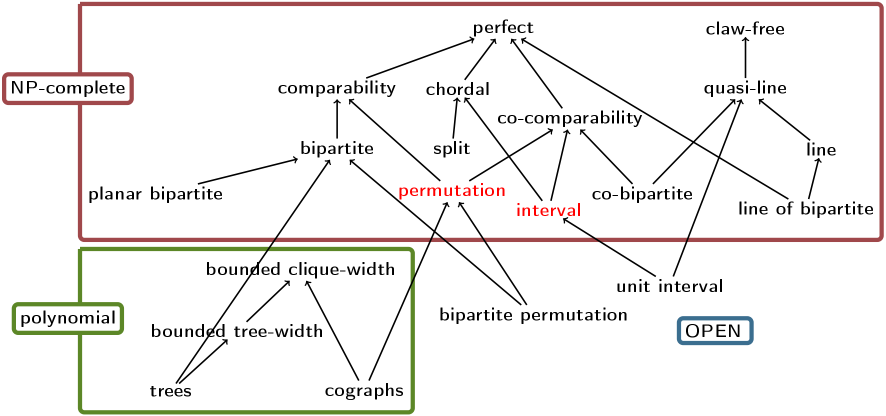


3DM instance on $3n$ elements, m triples.

$$\exists \text{ 3-dimensional matching} \iff \gamma^{\mathrm{ID}}(G) \leq 94m + 10n$$

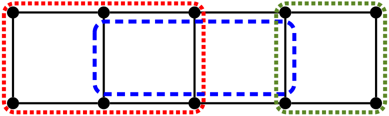


Complexity of IDENTIFYING CODE



Ladder graph L_m: grid graph $P_2 \square P_m$.

Cycle cover of graph G: set $\mathscr{S}$ of cycles of G s.t. $\bigcup_{S \in \mathscr{S}} E(S) = E(G)$.



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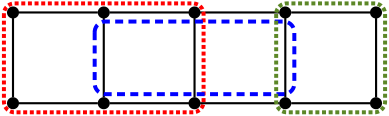

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INPUT: integer m, integer k, set $\mathscr{S}$ of cycles of L_m.
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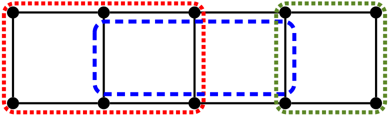

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IDENTIFYING CODE on unit interval graphs
reduces to LADDER CYCLE COVER.

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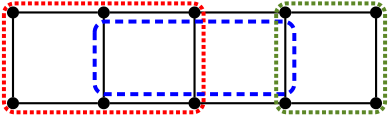

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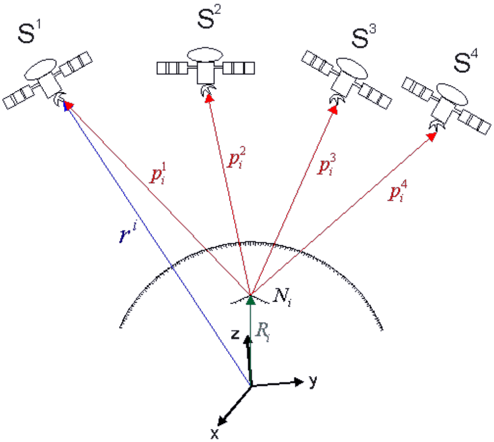
Question

Is LADDER CYCLE COVER polynomial-time solvable?

Part II: metric dimension

Determination of Position in 3D euclidean space

GPS: need to know the exact position of 4 satellites + distance to them



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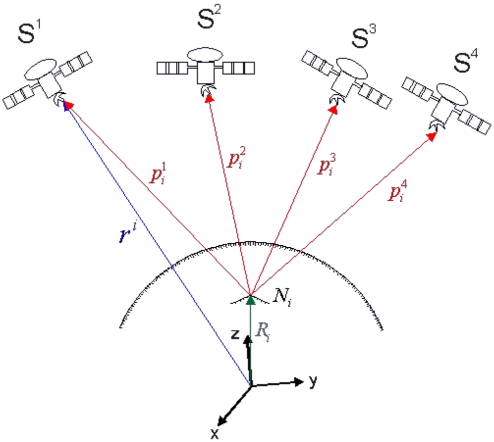

Question

Does the "GPS" approach also work in undirected unweighted graphs?

Now, $w \in V(G)$ separates $\{u, v\}$ if $dist(w, u) \neq dist(w, v)$

Definition - Resolving set (Slater, 1975 - Harary & Melter, 1976)

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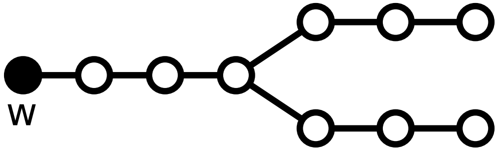

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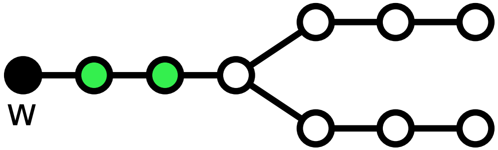

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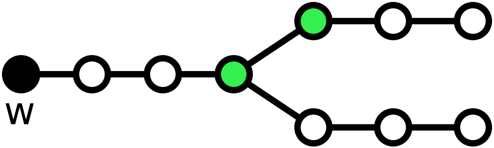

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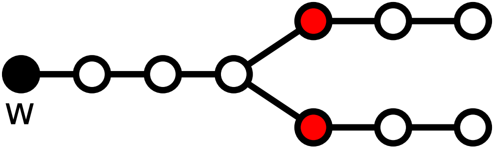

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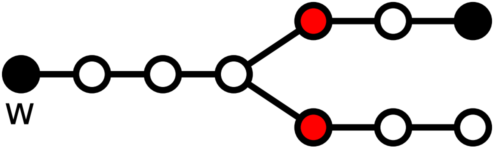

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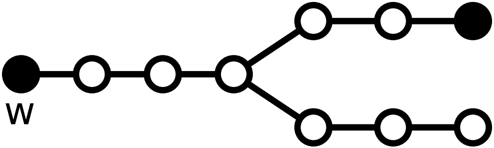

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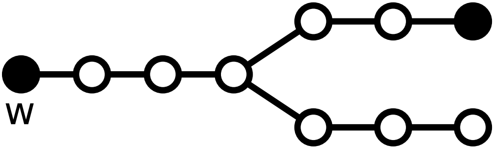


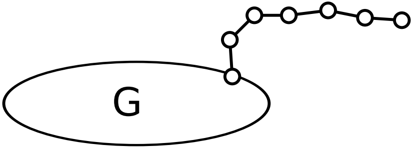
$MD(G)$: metric dimension of G, minimum size of a resolving set of G.

Proposition

$$MD(G) = 1 \Leftrightarrow G \text{ is a path}$$

Trees

Leg: path with all inner-vertices of degree 2, endpoints of degree ≥ 3 and 1.



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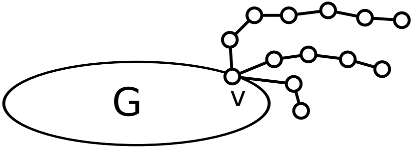

Observation

R resolving set. If v has k legs, at least $k-1$ legs contain a vertex of R.

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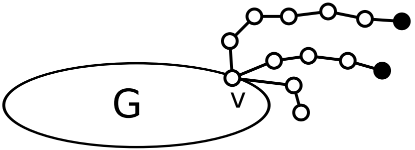

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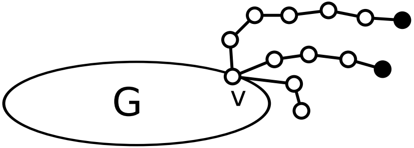

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Theorem (Slater 1975)

For any tree, the simple leg rule produces an optimal resolving set.

Bounds with diameter

Example of path: no bound $MD(G) \leq f(n)$ possible.

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G permutation graph or interval graph, $MD(G) = k$, diameter D. Then:

$$n = O(D^2 k^2).$$

Interval graphs:

- Interval in solution defines $\leq D+1$ "zones" (left and right) $\rightarrow k(D+1)$ zones
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Question

Is the bound tight? (There are interval graphs with $n = \Theta(Dk^2)$).

METRIC DIMENSION

INPUT: Graph G, integer k.
QUESTION: Is there a resolving set of G of size k?

- polynomial for:
  - trees (simple leg rule, Slater 1975)
  - outerplanar (Díaz, van Leeuwen, Pottonen, Serna, 2012)
  - bounded cyclomatic number (Epstein, Levin, Woeginger, 2012)
  - cographs (Epstein, Levin, Woeginger, 2012)

- NP-complete for:
  - general graphs (Garey & Johnson 1979)
  - planar (Díaz, van Leeuwen, Pottonen, Serna, 2012)
  - bipartite, co-bipartite, line, split (Epstein, Levin, Woeginger, 2012)
  - Gabriel unit disk (Hoffmann & Wanke 2012)

METRIC DIMENSION

INPUT: Graph G, integer k.
QUESTION: Is there a resolving set of G of size k?

- $O(\log n)$-approximable (SET COVER)
- hard to approximate within $o(\log n)$ for:
  - general graphs (Beerliova et al., 2006)
  - bipartite subcubic (Hartung & Nichterlein, 2013)
- APX-complete for graphs with min. degree $n - k$
  (Hauptmann, Schmied, Viehmann, 2012)

METRIC DIMENSION

INPUT: Graph G, integer k.
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W[2]-hard for parameter "solution size", even for bipartite subcubic graphs
(Hartung & Nichterlein, 2013)

G graph of diameter 2. S resolving set of G.

$\rightarrow$ Every vertex in $V(G) \setminus S$ is distiguished by its neighborhood within S

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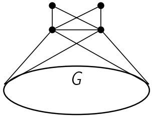
Reduction from LOCATING-DOMINATING SET to METRIC DIMENSION:

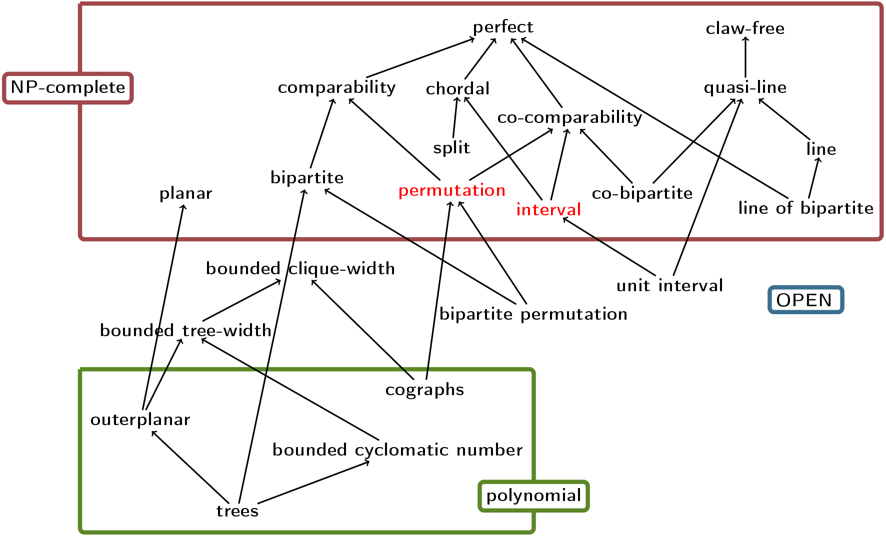


$k' = k + 2$

Theorem (F., Mertzios, Naserasr, Parreau, Valicov, 2014+)

METRIC DIMENSION is NP-complete for graphs that are both interval and permutation (and have diameter 2).

Complexity of METRIC DIMENSION



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(open for $k = 2$)

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Cyclomatic number of graph G (a.k.a feedback edge set number):
smallest k with a set S of k edges s.t. $G - S$ is a forest.

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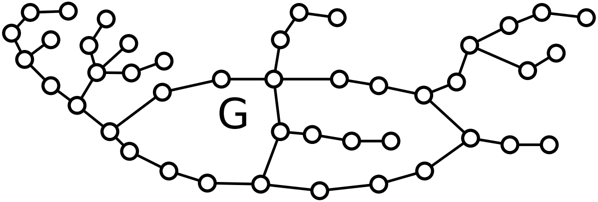
Complexity of METRIC DIMENSION for graphs of cyclomatic number k?

Recursive leg rule:

- Apply simple leg rule - if v has $k \geq 2$ legs, select $k-1$ leg endpoints.
- Prune graph by removing all legs with a selected vertex, replacing them by v.
- Apply recursively.



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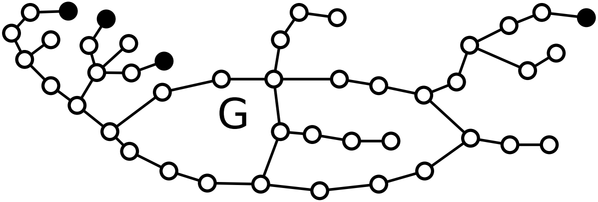

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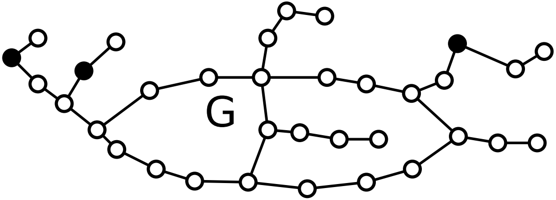

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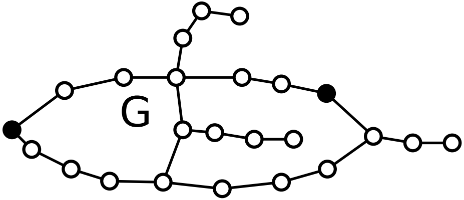

Bounded cyclomatic number - complexity

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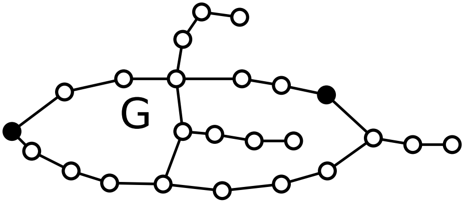

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Applying the recursive leg rule creates a computationally equivalent instance.

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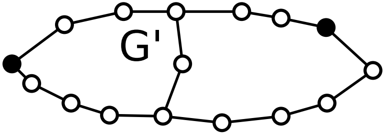

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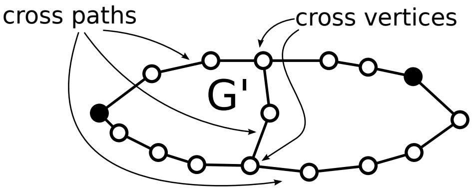

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cross vertex of G: degree ≥ 3 vertex in G'

cross path of G: thread between cross vertices

G has cyclomatic number k, reduced by recursive leg rule.

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G has at most $3k$ cross paths.

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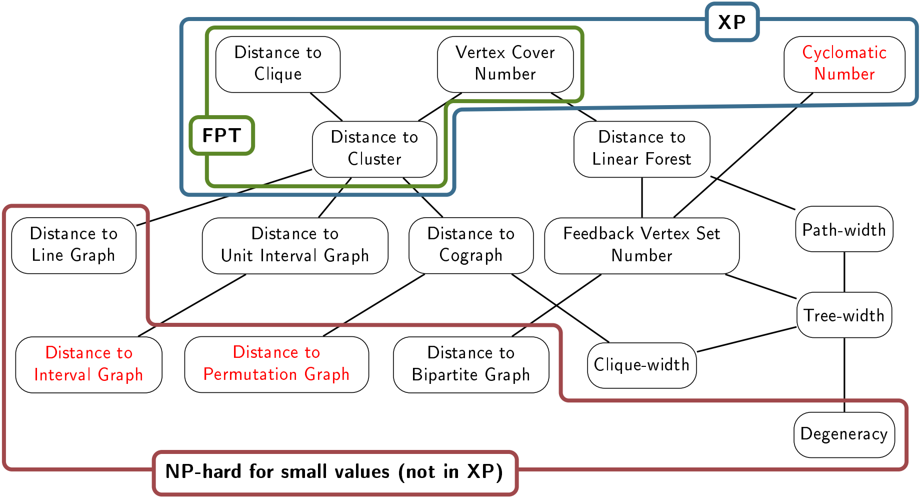
Proposition

- There is an $O(n^{9k})$ algorithm.
- There is a $9k$-approximation algorithm in polynomial time.
- There is a 3-approximation algorithm in FPT time $2^{3k} n^{O(1)}$.

METRIC DIMENSION: structural parameters



Perspectives

- Bounds for other classes? planar, unit disk, line, trapezoid, ...
- V-C dimension bound for metric dimension?
- Complexity of MD+ID for unit interval + bipartite permutation?
- Complexity of MD for bounded tree-width (and weaker parameters)?
- Parameterized complexity of MD (parameter "solution size")? interval, permutation, chordal, claw-free, planar...

THANKS FOR YOUR ATTENTION