Identification problems in graphs
Bounds and complexity

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joint work with:

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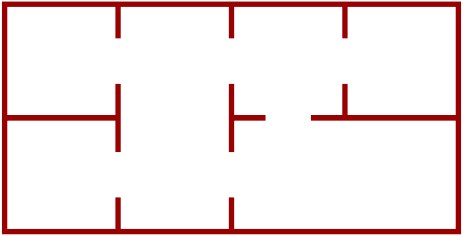
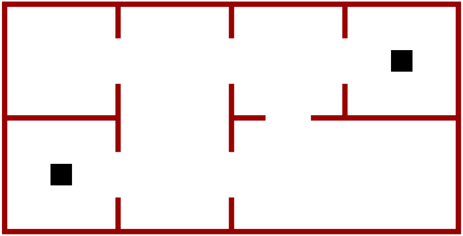
and
George Mertzios, Reza Naserasr, Aline Parreau, Petru Valicov

LIMOS, January 2015

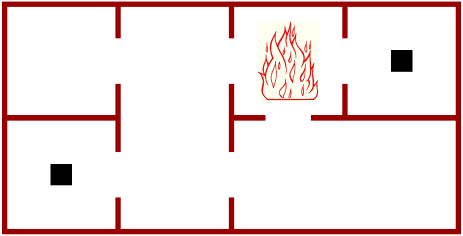
Part I: bounds for location-domination

Fire detection in a building





- Detector can detect fire in its room and its neighborhood (through a door).



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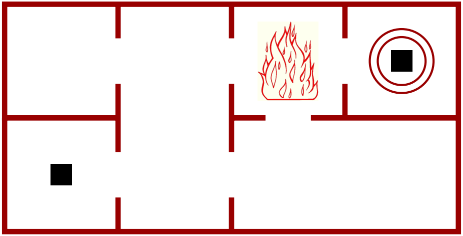

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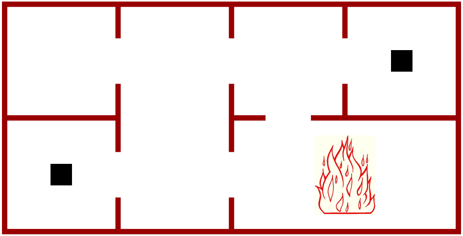

- Detector can detect fire in its room and its neighborhood (through a door).

Fire detection in a building

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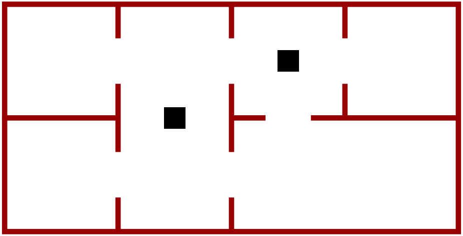
- Each room must contain a detector or have one in an adjacent room.

Fire detection in a building



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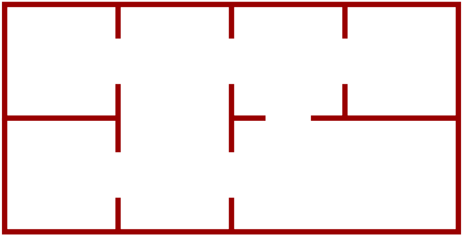
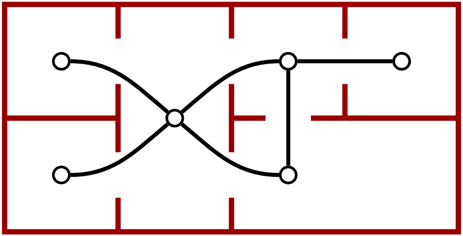


- Graph $G = (V, E)$. Vertices: rooms.
  Edges: between any two rooms connected by a door

Modelization with a graph



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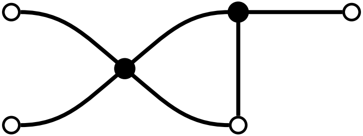

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- Set of detectors = dominating set $D \subseteq V$: $\forall u \in V, N[u] \cap D \neq \emptyset$

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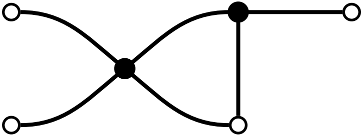

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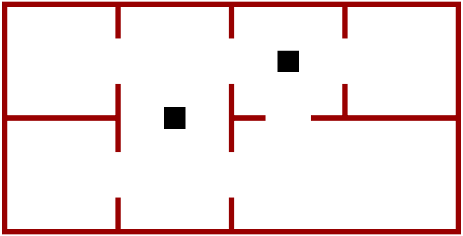
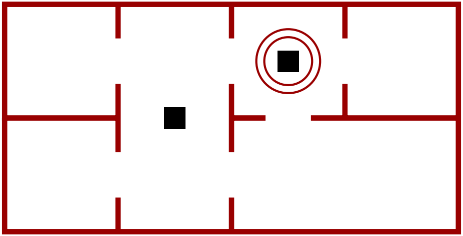
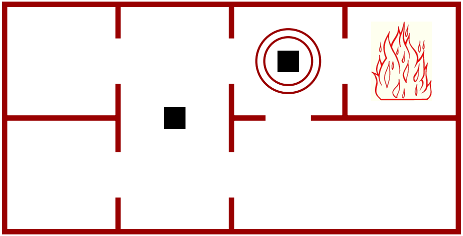
- Set of detectors = dominating set $D \subseteq V$: $\forall u \in V, N[u] \cap D \neq \emptyset$
- Domination number $\gamma(G)$: smallest size of a dominating set of G

Back to the building





Where is the fire ?



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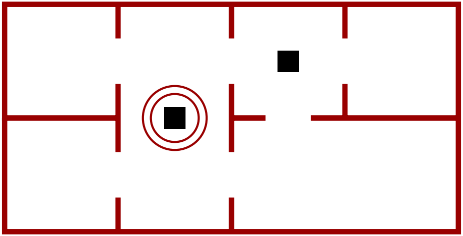

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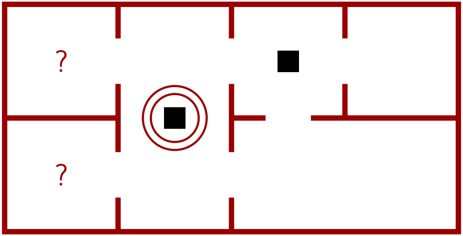
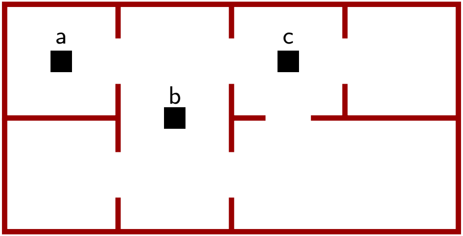
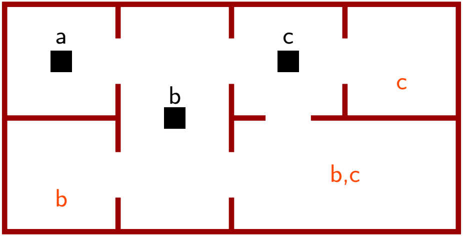


Where is the fire ?

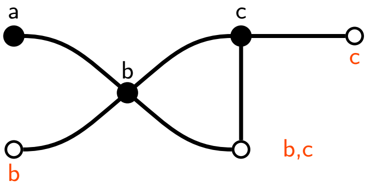
To locate the fire, we need more detectors.

Locating the fire



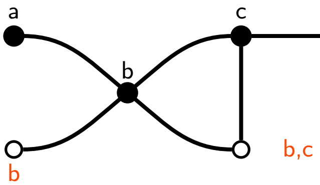


In each room with no detector, set of dominating detectors is distinct.



Peter Slater, 1980's. Locating-dominating set D:
subset of vertices of $G = (V, E)$ which is:

- dominating : $\forall u \in V, N[u] \cap D \neq \emptyset$,
- locating : $\forall u, v \in V \setminus D, N[u] \cap D \neq N[v] \cap D$.



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$\gamma_L(G)$: location-domination number of G,
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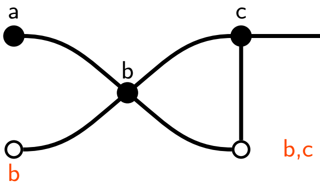

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Remark: $\gamma(G) \leq \gamma_L(G)$

Examples: paths

Domination number: $\gamma(P_n) = \lceil \frac{n}{3} \rceil$

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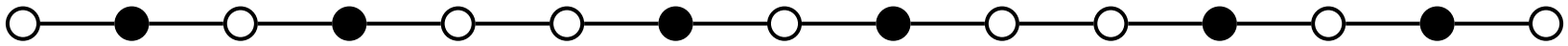
Location-domination number: $\gamma_L(P_n) = \lceil \frac{2n}{5} \rceil$



Upper bounds on the location-domination number

Theorem (Domination bound — Ore, 1960's)

G graph of order n, no isolated vertices. Then $\gamma(G) \leq \frac{n}{2}$.

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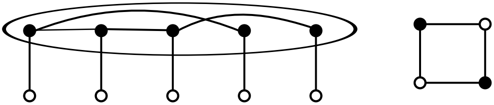
Tight examples:



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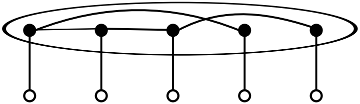
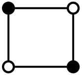
Tight examples:  

Theorem (Location-domination bound — Slater, 1980's)

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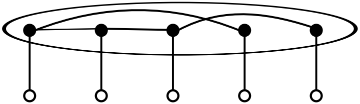
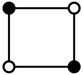
Tight examples:  

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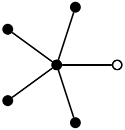
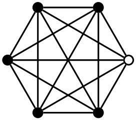
Tight examples:  

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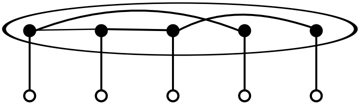
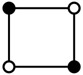
Tight examples:  

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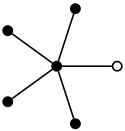
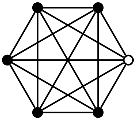
Tight examples:  

Remark: tight examples contain many twin-vertices!!

Upper bound - a conjecture

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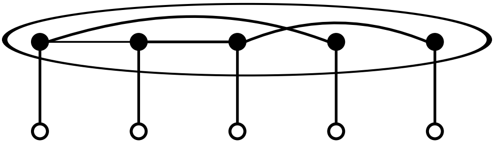
If true, tight: 1. domination-extremal graphs



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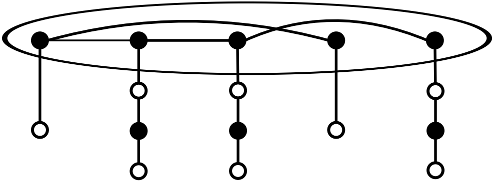
If true, tight: 2. a similar construction



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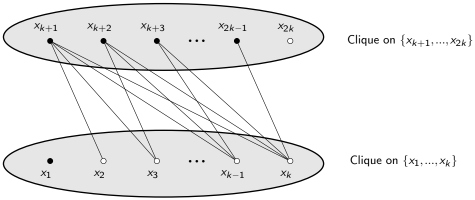
If true, tight: 3. a family with domination number 2



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Conjecture true if G has no 4-cycles, or if G is bipartite.

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Proof ideas:

- no 4-cycles: use a maximum matching
- bipartite: every vertex cover is a locating-dominating set

Upper bound - a conjecture

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Theorem (F., Henning, Löwenstein, Sasse, 2014+)

Conjecture true if G is split graph or complement of bipartite graph.

Theorem (F., Henning, 2014+)

Conjecture true if G is:
- cubic graph
- line graph

Split graph: clique + independent set

Cubic graph: all degrees equal to 3

Line graph: Intersection graph of the edges of a graph

Upper bound - a conjecture

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Remark: Nontrivial proofs using very different techniques!
→ Conjecture seems difficult.

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Lower bounds on the location-domination number

Lower bounds

Theorem (Slater, 1980's)

G graph of order n, $\gamma_L(G) = k$. Then $n \leq 2^k + k - 1$, i.e. $\gamma_L(G) = \Omega(\log n)$.

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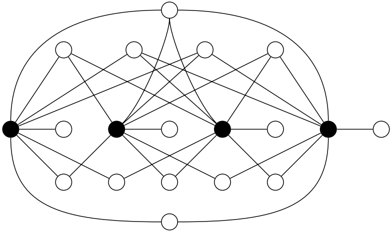
Tight example ($k = 4$):



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Theorem (Slater, 1980's)

G tree of order n, $\gamma_L(G)=k$. Then $n \leq 3k-1$, i.e. $\gamma_L(G) \geq \frac{n+1}{3}$.

Theorem (Rall & Slater, 1980's)

G planar graph, order n, $\gamma_L(G)=k$. Then $n \leq 7k-10$, i.e. $\gamma_L(G) \geq \frac{n+10}{7}$.

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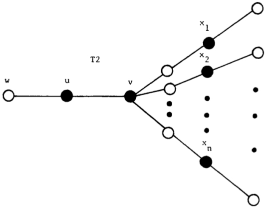


FIG. 2. Tree T2

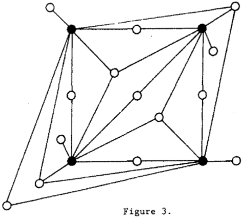
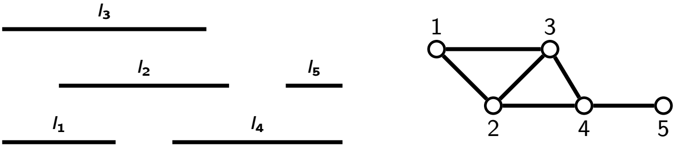


Figure 3.

Definition - Interval graph

Intersection graph of intervals of the real line.



Lower bound for interval graphs

Theorem (F., Mertzios, Naserasr, Parreau, Valicov, 2014+)

G interval graph of order n, $\gamma_L(G) = k$.

Then $n \leq \frac{k(k+3)}{2}$, i.e. $\gamma_L(G) = \Omega(\sqrt{n})$.

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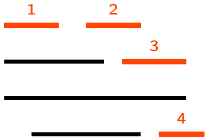

- Locating-dominating D of size k.
- Define zones using the right points of intervals in D.

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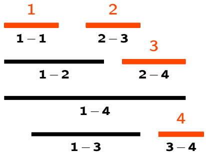

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- Each vertex intersects a consecutive set of intervals of D when ordered by left points.

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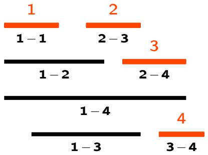

- Locating-dominating D of size k.
- Define zones using the right points of intervals in D.
- Each vertex intersects a consecutive set of intervals of D when ordered by left points.

$\rightarrow n \leq \sum_{i=1}^{k}(k-i) + k = \frac{k(k+3)}{2}$.

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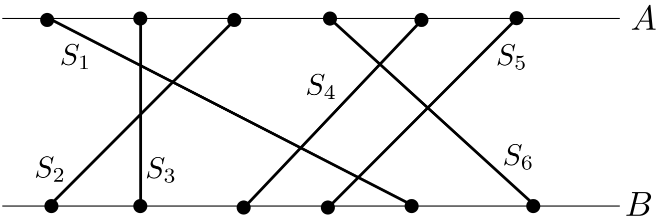
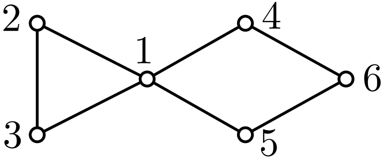
Then $n \leq \frac{k(k+3)}{2}$, i.e. $\gamma_L(G) = \Omega(\sqrt{n})$.

Tight:

Definition - Permutation graph

Given two parallel lines A and B:
intersection graph of segments joining A and B.





Lower bound for permutation graphs

Theorem (F., Mertzios, Naserasr, Parreau, Valicov, 2014+)

G permutation graph of order n, $\gamma_L(G) = k$.
Then $n \leq k^2 + k - 2$, i.e. $\gamma_L(G) = \Omega(\sqrt{n})$.

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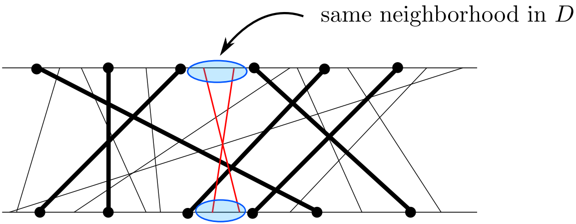

- Locating-sominating set D of size k: $k+1$ "top zones" and $k+1$ "bottom zones"
- Only one segment in $V \setminus D$ for one pair of zones

  $\rightarrow n \leq (k+1)^2 + k$
- Careful counting for the precise bound

Theorem (F., Mertzios, Naserasr, Parreau, Valicov, 2014+)

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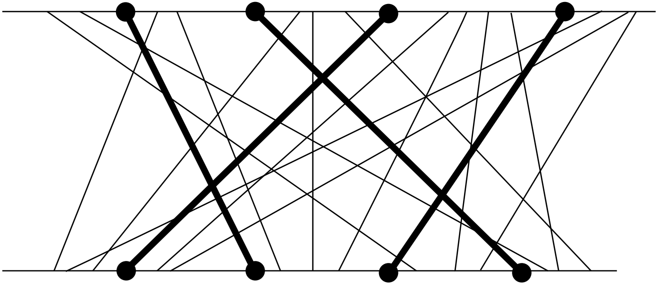

Theorem (F., Mertzios, Naserasr, Parreau, Valicov, 2014+)

Let G be a graph on n vertices, $\gamma_L(G) = k$.

- If G is *unit* interval, then $n \leq 3k - 1$.
- If G is *bipartite* permutation, then $n \leq 3k + 2$.
- If G is a cograph, then $n \leq 3k$.

Vapnis-Chervonenkis dimension

Set $X \subseteq V(G)$ is shattered:
for every subset $S \subseteq X$, there is a vertex v with $N[v] \cap X = S$

V-C dimension of G: maximum size of a shattered set in G

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Theorem (Bousquet, Lagoutte, Li, Parreau, Thomassé, 2014+)

G graph of order n, $\gamma_L(G) = k$, V-C dimension $\leq d$. Then $n = O(k^d)$.

$\rightarrow$ interval graphs ($d = 2$), line graphs ($d = 4$), permutation graphs ($d = 3$), unit disk graphs ($d = 3$), planar graphs ($d = 4$)...

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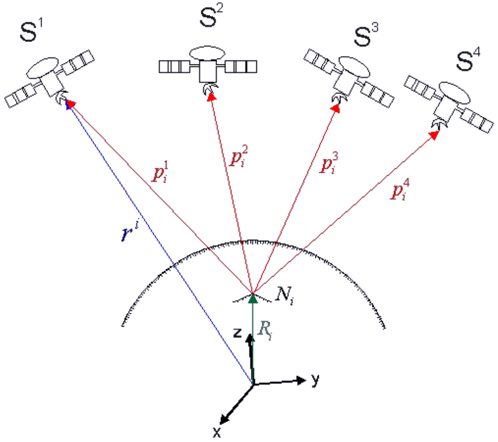
→ interval graphs ($d = 2$), line graphs ($d = 4$), permutation graphs ($d = 3$), unit disk graphs ($d = 3$), planar graphs ($d = 4$)...

But better bounds exist:

- planar: $n \leq 7k - 10$ (Slater & Rall, 1984)
- line: $n \leq \frac{8}{9}k^2$ (F., Gravier, Naserasr, Parreau, Valicov, 2013)
- permutation: $n \leq O(k^2)$ (F., Mertzios, Naserasr, Parreau, Valicov, 2014+)

Part II: metric dimension, bounds

GPS/GLONASS/Galileo/Beidou/IRNSS:
need to know the exact position of 4 satellites + distance to them



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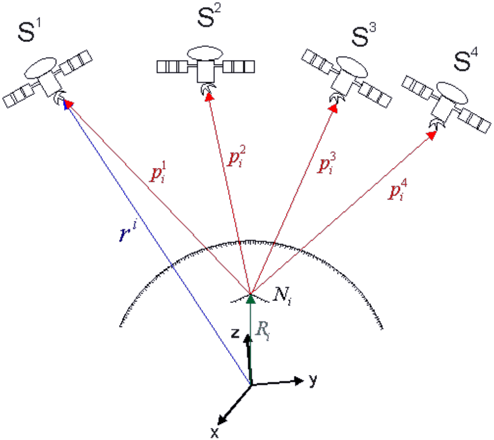

Question

Does the “GPS” approach also work in undirected unweighted graphs?

Metric dimension

Now, $w \in V(G)$ distinguishes $\{u,v\}$ if $dist(w,u) \neq dist(w,v)$

Definition - Resolving set (Slater, 1975 - Harary & Melter, 1976)

$R \subseteq V(G)$ resolving set of G:

$\forall u \neq v$ in $V(G)$, there exists $w \in R$ that distinguishes $\{u,v\}$.

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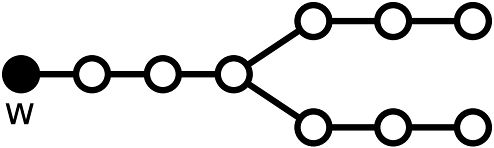

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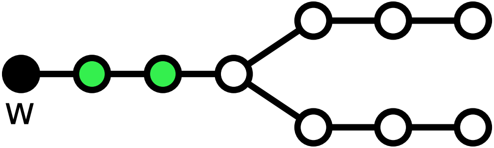

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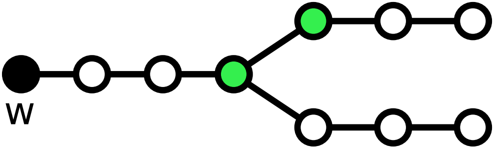

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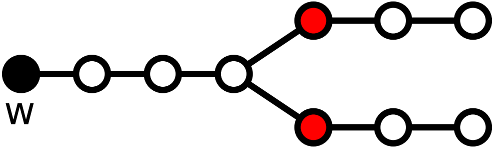

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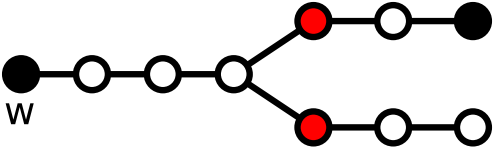

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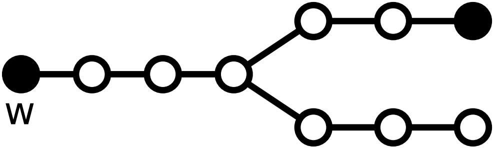

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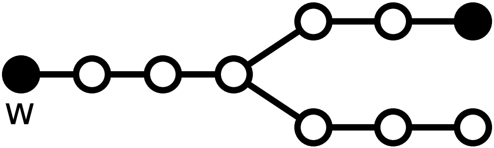

$MD(G)$: metric dimension of G, minimum size of a resolving set of G.

Remarks

Remark

- Any locating-dominating set is a resolving set, hence $MD(G) \leq \gamma_L(G)$.
- A locating-dominating set can be seen as a "distance-1-resolving set".

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- A locating-dominating set can be seen as a "distance-1-resolving set".

Proposition

$$MD(G) = 1 \Leftrightarrow G \text{ is a path}$$

Bounds with diameter

Example of path: no bound $n \leq f(MD(G))$ possible.

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Theorem (Khuller, Raghavachari & Rosenfeld, 2002)

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(diameter: maximum distance between two vertices)

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→ Proofs are similar as for locating-dominating sets.

→ Bounds are tight (up to constant factors).

Part III: Complexity and algorithms

LOCATING-DOMINATING SET

Complexity of LOCATING-DOMINATING SET

LOCATING-DOMINATING SET

INPUT: Graph G, integer k.
QUESTION: Is there a locating-dominating set of G of size k?

- polynomial for graphs of bounded cliquewidth via MSOL (Courcelle)
- NP-complete for:
  - bipartite (Charon, Hudry, Lobstein, 2003)
  - planar bipartite unit disk (Müller & Sereni, 2009)
  - planar arbitrary girth (Auger, 2010)
  - planar bipartite subcubic (F. 2013)
  - co-bipartite, split (F. 2013)
  - line (F., Gravier, Naserasr, Parreau, Valicov, 2013)

Complexity of LOCATING-DOMINATING SET

LOCATING-DOMINATING SET

INPUT: Graph G, integer k.
QUESTION: Is there a locating-dominating set of G of size k?

- $O(\log \Delta)$-approximable (SET COVER)
- constant c-approximation for:
  - planar, $c = 7$ (Slater, Rall, 1984)
  - line, $c = 4$ (F., Gravier, Naserasr, Parreau, Valicov, 2013)
  - interval, $c = 2$ (Bousquet, Lagoutte, Li, Parreau, Thomassé, 2014+)
  - unit interval, PTAS
- hard to approximate within $o(\log n)$ for:
  - general graphs (Laifenfeld, Trachtenberg + Suomela 2007)
  - bipartite, split, co-bipartite (F. 2013)
- APX-hard for:
  - line (F., Gravier, Naserasr, Parreau, Valicov, 2013)
  - subcubic bipartite (F. 2013)

LOCATING-DOMINATING SET

INPUT: Graph G, integer k.
QUESTION: Is there a locating-dominating set of G of size k?

- Trivially FPT for parameter k because $n \leq 2^k + k - 1$: whole graph is kernel!

$\longrightarrow n^{O(k)} = 2^{k^{O(k)}}$-time brute-force algorithm

Complexity - Interval and permutation graphs

Theorem (F., Mertzios, Naserasr, Parreau, Valicov, 2014+)

LOCATING-DOMINATING SET is NP-complete for graphs that are both interval and permutation.

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Reduction from 3-DIMENSIONAL MATCHING:

- INPUT: A, B, C sets and $\mathscr{T} \subset A \times B \times C$ triples
- QUESTION: is there a perfect 3-dimensional matching $M \subset T$, i.e., each element of $A \cup B \cup C$ appears exactly once in M?

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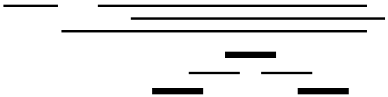
Main idea: an interval can separate pairs of intervals **far away** from each other (without affecting what lies in between)

Dominating gadget: ensure all intervals are dominated and most, separated.



Complexity - gadgets

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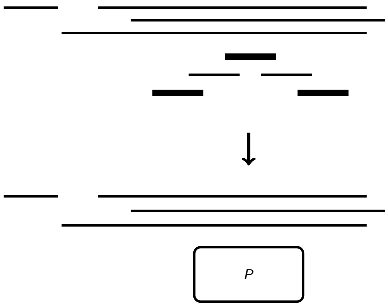
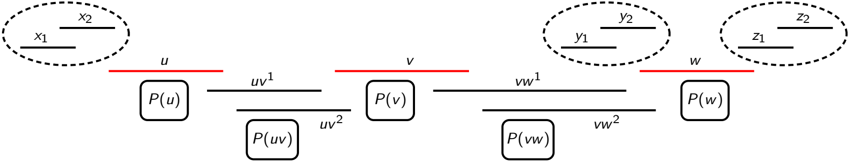


Transmitter gadget: to separate $\{uv^1, uv^2\}$ and $\{vw^1, vw^2\}$, either:

1. take only v into solution, or
2. take both u, w — and separate pairs $\{x_1, x_2\}$, $\{y_1, y_2\}$, $\{z_1, z_2\}$ "for free".



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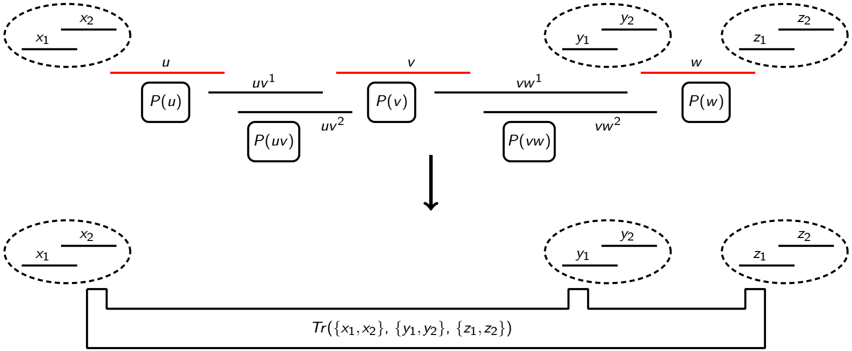
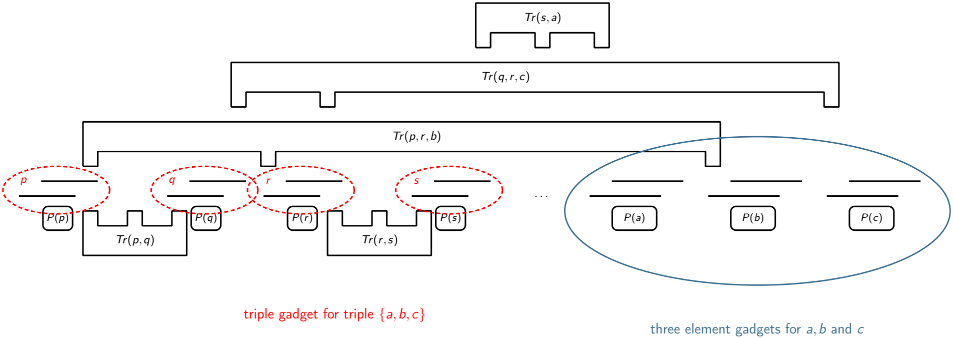
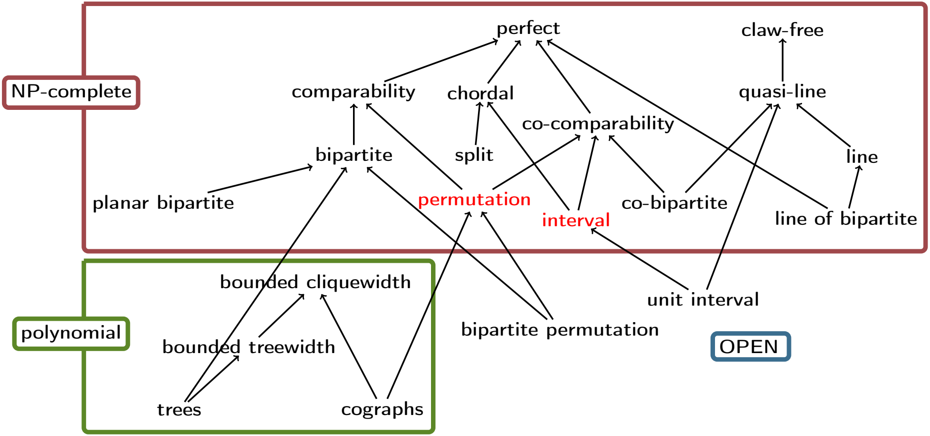


3DM instance on $3n$ elements, m triples.

$$\exists \text{ 3-dimensional matching} \iff \gamma_L(G) \leq 94m + 10n$$



Complexity of LOCATING-DOMINATING SET



METRIC DIMENSION

Complexity of METRIC DIMENSION

METRIC DIMENSION

INPUT: Graph G, integer k.
QUESTION: Is there a resolving set of G of size k?

- polynomial for:
  - trees (simple algorithm, Slater 1975)
  - outerplanar (Díaz, van Leeuwen, Pottonen, Serna, 2012)
  - bounded cyclomatic number (Epstein, Levin, Woeginger, 2012)
  - cographs (Epstein, Levin, Woeginger, 2012)

- NP-complete for:
  - general graphs (Garey & Johnson 1979)
  - planar (Díaz, van Leeuwen, Pottonen, Serna, 2012)
  - bipartite, co-bipartite, line, split (Epstein, Levin, Woeginger, 2012)
  - Gabriel unit disk (Hoffmann & Wanke 2012)

METRIC DIMENSION

INPUT: Graph G, integer k.
QUESTION: Is there a resolving set of G of size k?

- $O(\log n)$-approximable (SET COVER)
- hard to approximate within $o(\log n)$ for:
  - general graphs (Beerliova et al., 2006)
  - bipartite subcubic (Hartung & Nichterlein, 2013)

METRIC DIMENSION

INPUT: Graph G, integer k.
QUESTION: Is there a resolving set of G of size k?

W[2]-hard for parameter k, even for bipartite subcubic graphs
(Hartung & Nichterlein, 2013)
$\longrightarrow$ probably no $f(k)poly(n)$-time (FPT) algorithm

Interval and permutation graphs

G graph of diameter 2. S resolving set of G.

$\rightarrow$ Every vertex in $V(G) \setminus S$ is distiguished by its neighborhood within S

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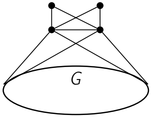
Reduction from LOCATING-DOMINATING SET to METRIC DIMENSION:

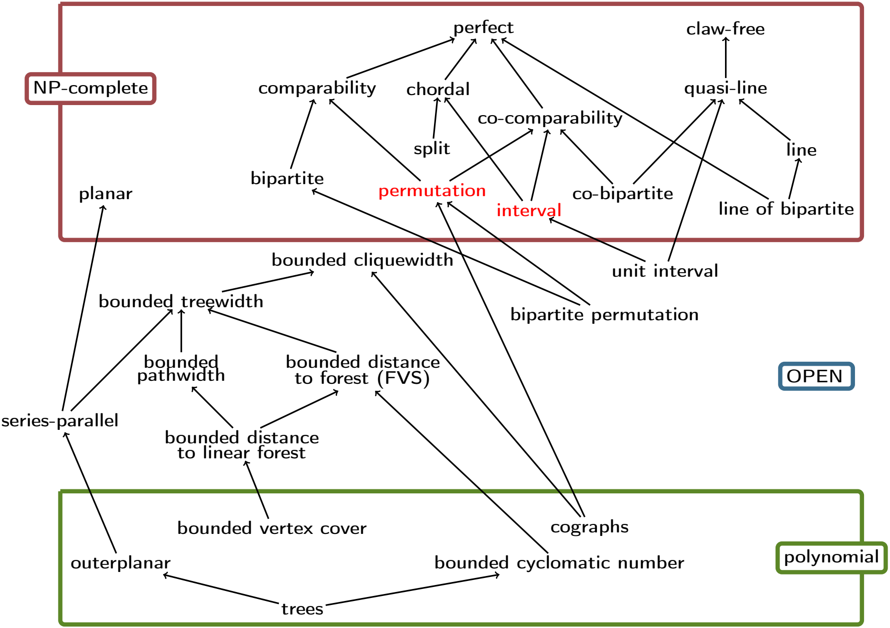


$$MD(G') = \gamma_L(G) + 2$$

Corollary (F., Mertzios, Naserasr, Parreau, Valicov, 2014+)

METRIC DIMENSION is NP-complete for graphs that are both interval and permutation (and have diameter 2).

Complexity of METRIC DIMENSION



An FPT algorithm for METRIC DIMENSION on interval graphs

Recall: METRIC DIMENSION W[2]-hard even for subcubic bipartite graphs
$\longrightarrow$ probably no $f(k)poly(n)$-time algorithm

Theorem (F., Mertzios, Naserasr, Parreau, Valicov, 2014+)

METRIC DIMENSION can be solved in time $2^{O(k^4)}n$ on interval graphs.

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METRIC DIMENSION can be solved in time $2^{O(k^4)}n$ on interval graphs.

Ideas:

- use dynamic programming on a path-decomposition of G^4.
- each bag has size $O(k^2)$.
- it suffices to separate vertices at distance 2
- "transmission" lemma for separation constraints

ONE MORE SLIDE

Open problems

- Solve the conjecture: $\gamma_L(G) \leq \frac{n}{2}$ if G twin-free?
- Investigate bounds for other "geometric" graphs, for MD and γ_L
- Complexity of LOCATING-DOMINATING SET, METRIC DIMENSION on unit interval graphs
- Complexity of METRIC DIMENSION for bounded treewidth
- Parameterized complexity of METRIC DIMENSION: planar graphs, chordal graphs, permutation graphs...

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THANKS FOR YOUR ATTENTION