Bounding the identifying code number of a graph using its degree parameters

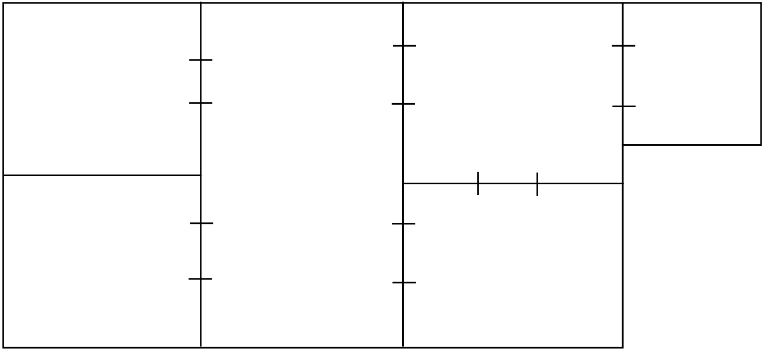
(a probabilistic approach)

Florent Foucaud (LaBRI)

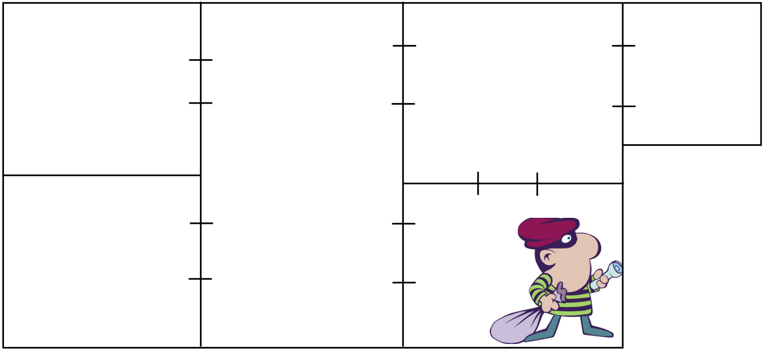
GT probas - January 9th, 2012

joint work with **Guillem Perarnau** (UPC, Barcelona)

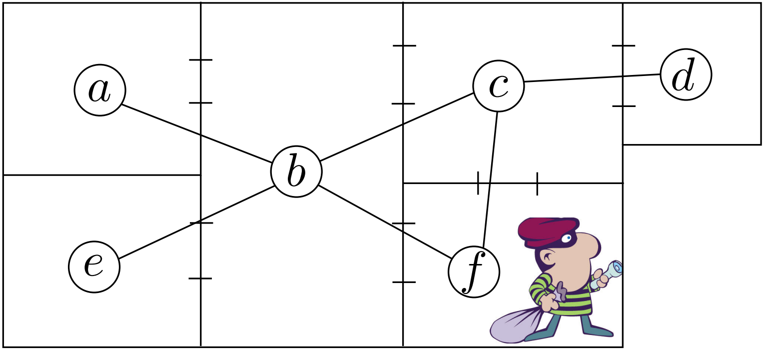
Locating a burglar in a museum



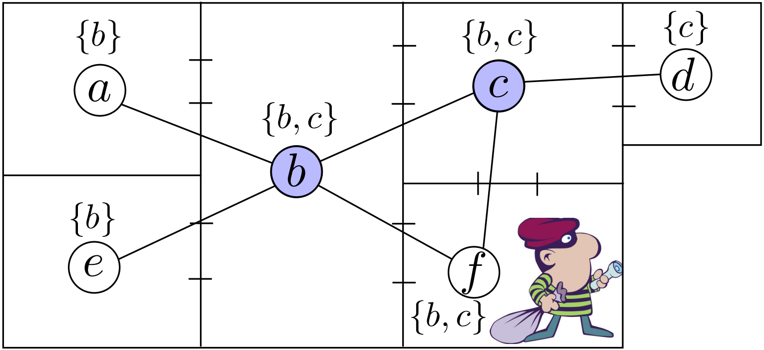
Locating a burglar in a museum



Locating a burglar in a museum

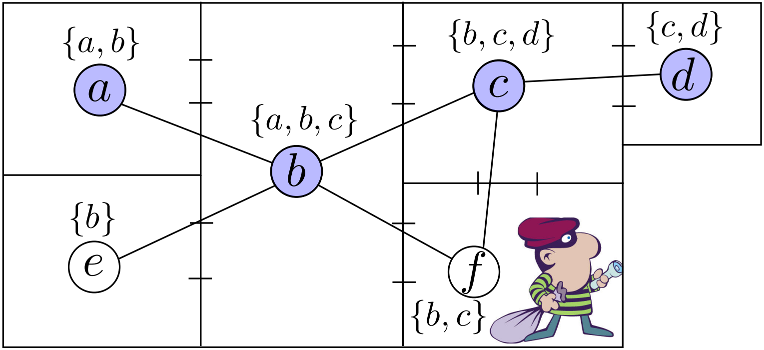


Graph $G = (V, E)$. V: vertices (rooms), $E \subseteq V \times V$: edges (doors)

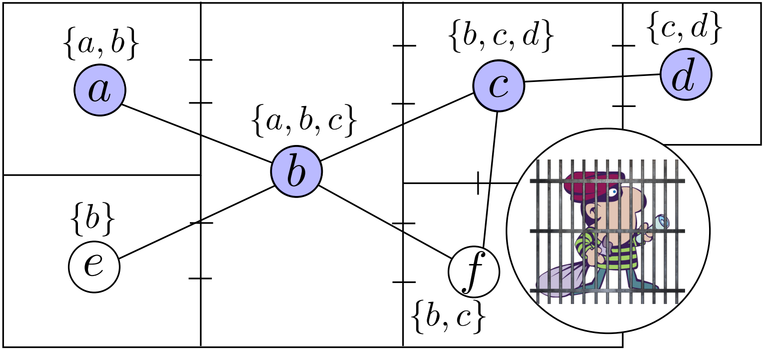


Graph $G = (V, E)$. V: vertices (rooms), $E \subseteq V \times V$: edges (doors)

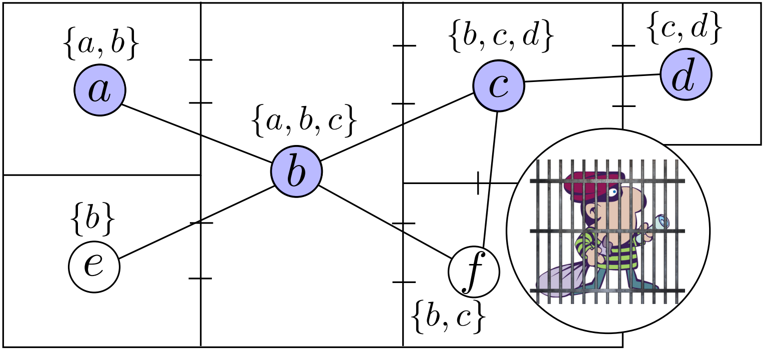
Locating a burglar in a museum



Graph $G=(V,E)$. V: vertices (rooms), $E \subseteq V \times V$: edges (doors)



Graph $G = (V, E)$. V: vertices (rooms), $E \subseteq V \times V$: edges (doors)



How many detectors do we need?

Let $N[u]$ be the set of vertices v s.t. $d(u,v) \leq 1$

Definition - Identifying code of G (Karpovsky, Chakrabarty, Levitin, 1998)

Subset C of V such that:

- C is a dominating set in G: $\forall u \in V$, $N[u] \cap C \neq \emptyset$, and
- C is a separating code in G: $\forall u \neq v$ of V, $N[u] \cap C \neq N[v] \cap C$
  Equivalently: $(N[u] \Delta N[v]) \cap C \neq \emptyset$ (covering symmetric differences)

Identifying codes: definition

Let $N[u]$ be the set of vertices v s.t. $d(u, v) \leq 1$

Definition - Identifying code of G (Karpovsky, Chakrabarty, Levitin, 1998)

Subset C of V such that:

- C is a dominating set in G: $\forall u \in V$, $N[u] \cap C \neq \emptyset$, and
- C is a separating code in G: $\forall u \neq v$ of V, $N[u] \cap C \neq N[v] \cap C$
  Equivalently: $(N[u] \Delta N[v]) \cap C \neq \emptyset$ (covering symmetric differences)

Notation - Identifying code number

$\gamma^{\text{ID}}(G)$: minimum cardinality of an identifying code of G

Identifying codes: definition

Let $N[u]$ be the set of vertices v s.t. $d(u, v) \leq 1$

Definition - Identifying code of G (Karpovsky, Chakrabarty, Levitin, 1998)

Subset C of V such that:

- C is a dominating set in G: $\forall u \in V$, $N[u] \cap C \neq \emptyset$, and
- C is a separating code in G: $\forall u \neq v$ of V, $N[u] \cap C \neq N[v] \cap C$
  Equivalently: $(N[u] \Delta N[v]) \cap C \neq \emptyset$ (covering symmetric differences)

Proposition

C is an identifying code IFF:

- C is a dominating set in G
- $\forall u \neq v$ of V **with** $d_G(u, v) \leq 2$, $(N[u] \Delta N[v]) \cap C \neq \emptyset$

Let $N[u]$ be the set of vertices v s.t. $d(u,v) \leq 1$

Remark

Not all graphs have an identifying code!

Twins = pair u, v such that $N[u] = N[v]$.

A graph is identifiable iff it is twin-free (i.e. it has no twins).

Identifiable graphs

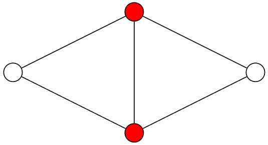
Let $N[u]$ be the set of vertices v s.t. $d(u, v) \leq 1$

Remark

Not all graphs have an identifying code!

Twins = pair u, v such that $N[u] = N[v]$.

A graph is identifiable iff it is twin-free (i.e. it has no twins).



Identifiable graphs

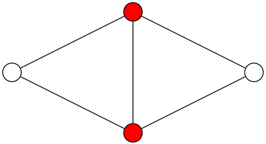
Let $N[u]$ be the set of vertices v s.t. $d(u, v) \leq 1$

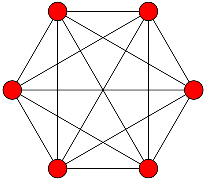
Remark

Not all graphs have an identifying code!

Twins = pair u, v such that $N[u] = N[v]$.

A graph is identifiable iff it is twin-free (i.e. it has no twins).

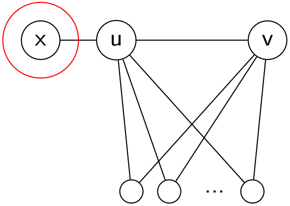




Forced vertices

u, v such that $N[v] \Delta N[u] = \{x\}$

Then $x \in C$, forced by uv.



Notation

Let $NF(G)$ be the proportion of non forced vertices of G

$$NF(G) = \frac{\#\text{non-forced vertices in G}}{\#\text{vertices in G}}$$

Degree parameters of a graph

Graph $G = (V, E)$, vertex $v \in V$.

- degree of v: number of edges it is incident to
- maximum degree d of G: max. degree of a vertex in G
- d-regular graph: all vertices have degree d

Previous results (1/2)

Theorem (Karpovsky, Chakrabarty, Levitin, 1998 + Gravier, Moncel, 2007)

Let G be an identifiable graph with at least one edge, then

$$\lceil \log_2(n+1) \rceil \leq \gamma^{\text{ID}}(G) \leq n-1$$

Previous results (1/2)

Theorem (Karpovsky, Chakrabarty, Levitin, 1998 + Gravier, Moncel, 2007)

Let G be an identifiable graph with at least one edge, then

$$\lceil \log_2(n+1) \rceil \leq \gamma^{\mathrm{ID}}(G) \leq n-1$$

Theorem (Karpovsky, Chakrabarty, Levitin, 1998)

Let G be an identifiable graph with maximum degree d, then

$$\frac{2n}{d+2} \leq \gamma^{\mathrm{ID}}(G)$$

Previous results (1/2)

Theorem (Karpovsky, Chakrabarty, Levitin, 1998 + Gravier, Moncel, 2007)

Let G be an identifiable graph with at least one edge, then

$$\lceil \log_2(n+1) \rceil \leq \gamma^{\text{ID}}(G) \leq n-1$$

Theorem (Karpovsky, Chakrabarty, Levitin, 1998)

Let G be an identifiable graph with maximum degree d, then

$$\frac{2n}{d+2} \leq \gamma^{\text{ID}}(G)$$

Conjecture (F., Klasing, Kosowski, Raspaud, 2009+)

Let G be a connected nontrivial identifiable graph of max. degree d. Then

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{d} + c \text{ for some constant } c$$

Previous results (1/2)

Theorem (Karpovsky, Chakrabarty, Levitin, 1998 + Gravier, Moncel, 2007)

Let G be an identifiable graph with at least one edge, then

$$\lceil \log_2(n+1) \rceil \leq \gamma^{\text{ID}}(G) \leq n-1$$

Theorem (Karpovsky, Chakrabarty, Levitin, 1998)

Let G be an identifiable graph with maximum degree d, then

$$\frac{2n}{d+2} \leq \gamma^{\text{ID}}(G)$$

Conjecture (F., Klasing, Kosowski, Raspaud, 2009+)

Let G be a connected nontrivial identifiable graph of max. degree d. Then

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{d} + c \text{ for some constant } c$$

This would be tight. True for $d = 2$ ($c = 3/2$) and $d = n-1$ ($c = 1$).

Extremal examples

Conjecture (F., Klasing, Kosowski, Raspaud, 2009+)

Let G be a connected nontrivial identifiable graph of max. degree d. Then

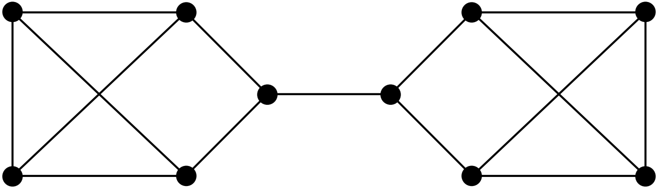
$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{d} + c \text{ for some constant } c$$

Extremal examples

Conjecture (F., Klasing, Kosowski, Raspaud, 2009+)

Let G be a connected nontrivial identifiable graph of max. degree d. Then

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{d} + c \text{ for some constant } c$$

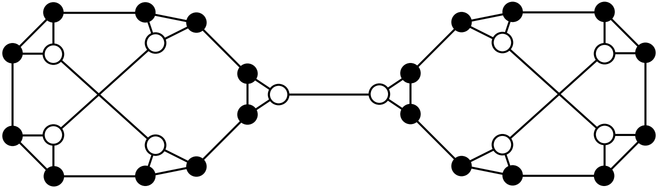


Extremal examples

Conjecture (F., Klasing, Kosowski, Raspaud, 2009+)

Let G be a connected nontrivial identifiable graph of max. degree d. Then

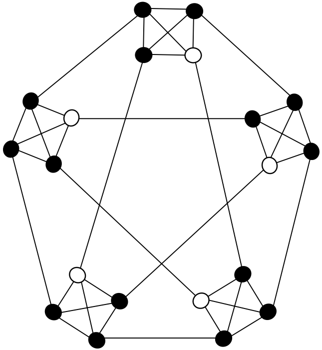
$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{d} + c \text{ for some constant } c$$



Conjecture (F., Klasing, Kosowski, Raspaud, 2009+)

Let G be a connected nontrivial identifiable graph of max. degree d. Then

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{d} + c \text{ for some constant } c$$

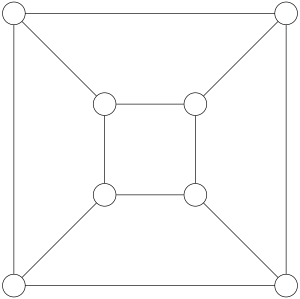


Extremal examples

Conjecture (F., Klasing, Kosowski, Raspaud, 2009+)

Let G be a connected nontrivial identifiable graph of max. degree d. Then

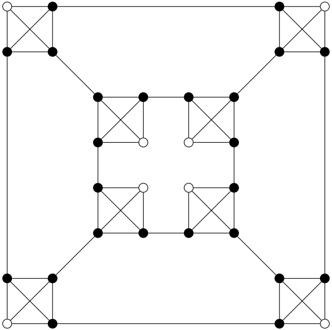
$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{d} + c \text{ for some constant } c$$



Conjecture (F., Klasing, Kosowski, Raspaud, 2009+)

Let G be a connected nontrivial identifiable graph of max. degree d. Then

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{d} + c \text{ for some constant } c$$



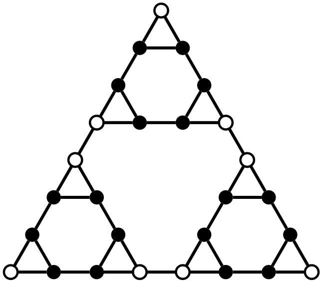
Extremal examples

Conjecture (F., Klasing, Kosowski, Raspaud, 2009+)

Let G be a connected nontrivial identifiable graph of max. degree d. Then

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{d} + c \text{ for some constant } c$$

Also: Sierpiński graphs
(see A. Parreau, S. Gravier, M. Kovše, M. Mollard and J. Moncel, 2011+)



Conjecture (F., Klasing, Kosowski, Raspaud, 2009+)

Let G be a connected nontrivial identifiable graph of max. degree d. Then

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{d} + c \text{ for some constant } c$$

Question

Can we prove that $\gamma^{\text{ID}}(G) \leq n - \frac{n}{\Theta(d)}$?

Previous results (2/2)

Conjecture (F., Klasing, Kosowski, Raspaud, 2009+)

Let G be a connected nontrivial identifiable graph of max. degree d. Then

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{d} + c \text{ for some constant } c$$

Theorem (F., Guerrini, Kovse, Naserasr, Parreau, Valicov, 2011)

Let G be a connected identifiable graph of maximum degree d. Then

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{\Theta(d^5)}$$

If G has no forced vertices, $\gamma^{\text{ID}}(G) \leq n - \frac{n}{\Theta(d^3)}$

Previous results (2/2)

Conjecture (F., Klasing, Kosowski, Raspaud, 2009+)

Let G be a connected nontrivial identifiable graph of max. degree d. Then

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{d} + c \text{ for some constant } c$$

Theorem (F., Guerrini, Kovse, Naserasr, Parreau, Valicov, 2011)

Let G be a connected identifiable graph of maximum degree d. Then

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{\Theta(d^5)}$$

If G has no forced vertices, $\gamma^{\text{ID}}(G) \leq n - \frac{n}{\Theta(d^3)}$

Theorem (F., Klasing, Kosowski, Raspaud, 2009+)

Let G be a connected identifiable triangle-free graph of max. degree d. Then

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{d(1+o_d(1))}$$

If G is bipartite, $\gamma^{\text{ID}}(G) \leq n - \frac{n}{d+9}$

The probabilistic method

Technique initiated, among others, by Pál Erdős
used mainly in combinatorics (Ramsey theory, graph theory, ...)

1. Define a suitable probability space
2. Select some object from this space using randomness
3. Prove that with nonzero probability, certain "good" conditions hold
4. Conclusion: there always exists a "good" object

Classic reference: Noga Alon and Joel Spencer, *The probabilistic method*

Upper bounds for $\gamma^{ID}(G)$

Notation

Let $NF(G)$ be the proportion of non forced vertices of G

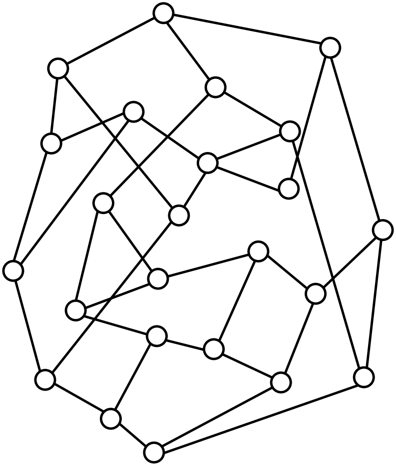
$$NF(G) = \frac{\#\text{non-forced vertices in G}}{\#\text{vertices in G}}$$

Theorem (F., Perarnau, 2011+)

There exists an integer d_0 such that for each identifiable graph G on n vertices having maximum degree $d \geq d_0$ and no isolated vertices,

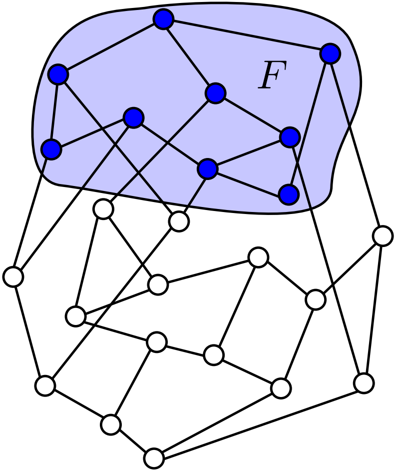
$$\gamma^{ID}(G) \leq n - \frac{n \cdot NF(G)^2}{85d}$$

Proof - select a set at random...



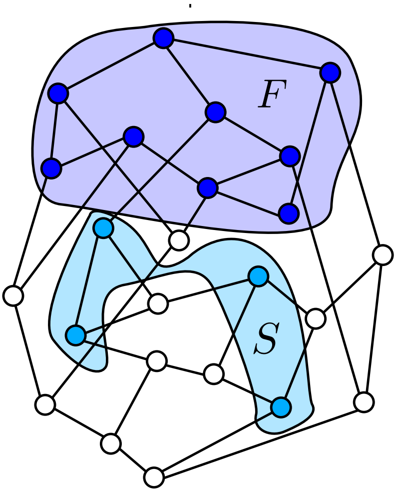
Proof - select a set at random...

- F: forced vertices



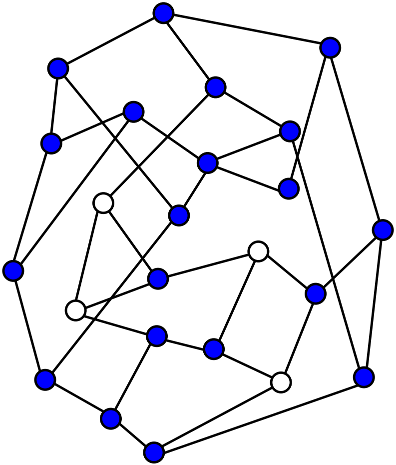
- F: forced vertices

- Select a random set S from $V' = V \setminus F$: each vertex $v \in S$ with prob. p.



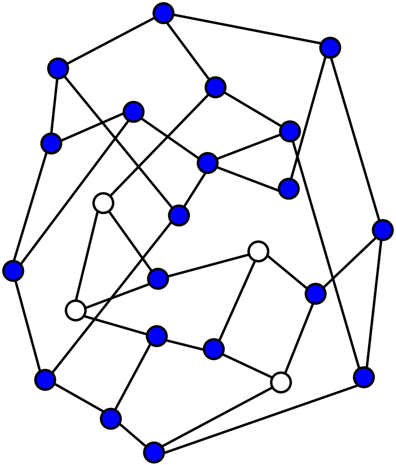
Proof - select a set at random...

- F: forced vertices
- Select a random set S from $V' = V \setminus F$: each vertex $v \in S$ with prob. p.
- $\mathcal{C} = V \setminus S$



Proof - select a set at random...

- F: forced vertices
- Select a random set S from $V' = V \setminus F$: each vertex $v \in S$ with prob. p.
- $\mathcal{C} = V \setminus S$



Proof - Lovász Local Lemma

Theorem (Weighted Local Lemma: particular case of the Local Lemma Erdős, Lovász, 1973 - Molloy, Reed, 2001[1])

Let $0 < p \leq \frac{1}{4}$ and $\mathcal{E} = \{E_1, \ldots, E_M\}$ be a set of "bad" events such that each E_i is mutually independent of $\mathcal{E} \setminus (\mathcal{D}_i \cup \{E_i\})$ where $\mathcal{D}_i \subseteq \mathcal{E}$, and

- $Pr(E_i) \leq p^{t_i}$
- $\sum_{E_j \in D_i} (2p)^{t_j} \leq \frac{t_i}{2}$

Then $Pr(\bigcap_{i=1}^{M} \overline{E_i}) \geq \prod_{i=1}^{M}(1 - (2p)^{t_i}) \geq \exp\left\{-2 \log 2 \sum_{i=1}^{m} (2p)^{t_i}\right\} > 0.$

1: Molloy and Reed - *Graph colouring and the probabilistic method*, 2001

Proof - Lovász Local Lemma

Theorem (Weighted Local Lemma: particular case of the Local Lemma Erdős, Lovász, 1973 - Molloy, Reed, 2001[1])

Let $0 < p \leq \frac{1}{4}$ and $\mathcal{E} = \{E_1, \ldots, E_M\}$ be a set of "bad" events such that each E_i is mutually independent of $\mathcal{E} \setminus (\mathcal{D}_i \cup \{E_i\})$ where $\mathcal{D}_i \subseteq \mathcal{E}$, and

- $Pr(E_i) \leq p^{t_i}$
- $\sum_{E_j \in D_i} (2p)^{t_j} \leq \frac{t_i}{2}$

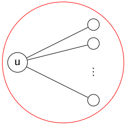
Then $Pr(\bigcap_{i=1}^{M} \overline{E_i}) \geq \prod_{i=1}^{M} (1 - (2p)^{t_i}) \geq \exp\left\{-2 \log 2 \sum_{i=1}^{m} (2p)^{t_i}\right\} > 0.$

$\Longrightarrow$ If the dependencies are "rare":

with non-zero probability none of the bad events occur

1: Molloy and Reed - *Graph colouring and the probabilistic method*, 2001

Set the bad events...

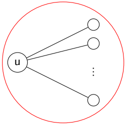


$p^{d+1} \leq Pr(A_u) \leq p^2$

Event A_u

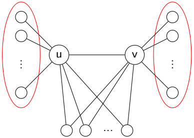
Size t_i of an event E_i: number of vertices circled in red

Event of type T^j ($T \in \{A, B, D\}$): event of type T with size j.



$$p^{d+1} \leq Pr(A_u) \leq p^2$$

Event A_u



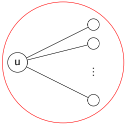
Event $B_{u,v}$

$$p^{2d-2} \leq Pr(B_{u,v}) \leq p^2$$

Size t_i of an event E_i: number of vertices circled in red

Event of type T^j ($T \in \{A, B, D\}$): event of type T with size j.

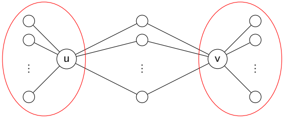
Set the bad events...



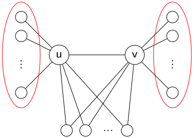
$p^{d+1} \leq Pr(A_u) \leq p^2$

Event A_u

$p^{2d} \leq Pr(C_{u,v}) \leq p^4$



Event $C_{u,v}$



Event $B_{u,v}$

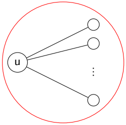
$p^{2d-2} \leq Pr(B_{u,v}) \leq p^2$

Size t_i of an event E_i: number of vertices circled in red

Event of type T^j ($T \in \{A, B, D\}$): event of type T with size j.

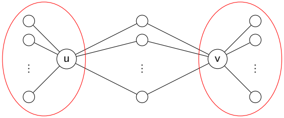
Set the bad events...

$p^{d+1} \leq Pr(A_u) \leq p^2$

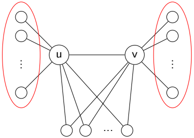


Event A_u

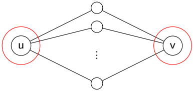
$p^{2d} \leq Pr(C_{u,v}) \leq p^4$



Event $C_{u,v}$



Event $B_{u,v}$



$Pr(D_{u,v}) = p^2$

Event $D_{u,v}$

$p^{2d-2} \leq Pr(B_{u,v}) \leq p^2$

Size t_i of an event E_i: number of vertices circled in red

Event of type T^j ($T \in \{A, B, D\}$): event of type T with size j.

Proof (regular case) - Lovász Local Lemma, technicalities

Theorem (Weighted Local Lemma)

Bad events: $\mathcal{E} = \{E_1, \ldots, E_M\}$.

- $Pr(E_i) \leq p^{t_i}$
- $\sum_{E_j \in D_i} (2p)^{t_j} \leq \frac{t_i}{2}$

Then $Pr(\bigcap_{i=1}^{M} \overline{E_i}) \geq \prod_{i=1}^{M} (1 - (2p)^{t_i}) > 0$.

Compute "intersection" between events (on board)

Taking $p = \frac{1}{kd} \Longrightarrow$ LLL can be applied

Proof (regular case) - the set can be small...

By the LLL we know that

There exists some set S *with* $\mathbb{E}(|S|) = \frac{nNF(G)}{k \cdot d}$ *such that no bad event occurs*

Proof (regular case) - the set can be small...

By the LLL we know that

There exists some set S with $\mathbb{E}(|S|) = \frac{nNF(G)}{k \cdot d}$ such that no bad event occurs

And we also know: if $S = \emptyset$, $\mathcal{C} = V \setminus S = V$ is a trivial code!

Proof (regular case) - the set can be small...

By the LLL we know that

There exists some set S with $\mathbb{E}(|S|) = \frac{nNF(G)}{k \cdot d}$ such that no bad event occurs

And we also know: if $S = \emptyset$, $\mathcal{C} = V \setminus S = V$ is a trivial code!

But by the LLL we know more:

$$\Pr\left(\bigcap_{i=1}^{m} \overline{E_i}\right) > \exp\left\{-2\log 2 \sum_{i=1}^{m} (2p)^{t_i}\right\}$$

Proof (regular case) - the set can be small...

By the LLL we know that

There exists some set S with $\mathbb{E}(|S|) = \frac{nNF(G)}{k \cdot d}$ such that no bad event occurs

And we also know: if $S = \emptyset$, $\mathcal{C} = V \setminus S = V$ is a trivial code!

But by the LLL we know more:

$$\Pr\left(\bigcap_{i=1}^{m} \overline{E_i}\right) > \exp\left\{-\frac{17\log 2}{2k^2 d} n\right\}$$

The probability to have a good set S is at least $\exp\left\{-\frac{17\log 2}{2k^2 d} n\right\}$

Proof (regular case) - concentration inequality

Theorem (Chernoff bound)

Let $X_1, \ldots, X_m$ a set of i.i.d random variables s.t. $Pr(X_i = 1) = p$ and $Pr(X_i = 0) = 1 - p$ and $X = \sum X_i$. Then

$$\Pr(\mathbb{E}(X) - X > \alpha) \leq \exp\left\{-\frac{\alpha^2}{2mp}\right\}$$

Proof (regular case) - concentration inequality

Theorem (Chernoff bound)

Let $X_1, \ldots, X_m$ a set of i.i.d random variables s.t. $Pr(X_i = 1) = p$ and $Pr(X_i = 0) = 1 - p$ and $X = \sum X_i$. Then

$$\Pr(\mathbb{E}(X) - X > \alpha) \leq \exp\left\{-\frac{\alpha^2}{2mp}\right\}$$

For each $v_i \in V \setminus F$ define the random variable:

$$X_i = \begin{cases} 1 & \text{if } v_i \in C \\ 0 & \text{otherwise} \end{cases}$$

Then, we set $\alpha = \frac{nNF(G)}{cd}$. Using $mp = \frac{nNF(G)}{kd}$:

$$\Pr\left(\mathbb{E}(X) - X > \frac{nNF(G)}{cd}\right) \leq \exp\left\{\frac{kNF(G)}{2c^2d}n\right\}$$

Theorem (Chernoff bound)

Let $X_1, \ldots, X_m$ a set of i.i.d random variables s.t. $Pr(X_i = 1) = p$ and $Pr(X_i = 0) = 1 - p$ and $X = \sum X_i$. Then

$$\Pr(\mathbb{E}(X) - X > \alpha) \leq \exp\left\{-\frac{\alpha^2}{2mp}\right\}$$

For each $v_i \in V \setminus F$ define the random variable:

$$X_i = \begin{cases} 1 & \text{if } v_i \in C \\ 0 & \text{otherwise} \end{cases}$$

Then, we set $\alpha = \frac{nNF(G)}{cd}$. Using $mp = \frac{nNF(G)}{kd}$:

$$\Pr\left(\mathbb{E}(X) - X > \frac{nNF(G)}{cd}\right) \leq \exp\left\{\frac{kNF(G)}{2c^2 d} n\right\}$$

Probability that S is too small: at most $\exp\left\{-\frac{kNF(G)}{2c^2 d} n\right\}$

Proof (regular case) - size of the code

$$\Pr(S \text{ good}) - \Pr(S \text{ too small}) > 0$$

Proof (regular case) - size of the code

$$\Pr(S \text{ good}) - \Pr(S \text{ too small}) > 0$$

There exist S such that $V \setminus S$ is an identifying code

$$|S| = X \geq \mathbb{E}(X) - \frac{nNF(G)}{cd} = \frac{nNF(G)}{kd} - \frac{nNF(G)}{cd} \geq ... \geq \frac{nNF(G)^2}{85d}$$

Proof (regular case) - size of the code

$$\Pr(S \text{ good}) - \Pr(S \text{ too small}) > 0$$

There exist S such that $V \setminus S$ is an identifying code

$$|S| = X \geq \mathbb{E}(X) - \frac{nNF(G)}{cd} = \frac{nNF(G)}{kd} - \frac{nNF(G)}{cd} \geq \ldots \geq \frac{nNF(G)^2}{85d}$$

$$|\mathcal{C}| = |V \setminus S| \leq n - \frac{nNF(G)^2}{85d}$$

Bounding the number of forced vertices

Proposition

Let $NF(G)$ be the proportion of non forced vertices of G. Then

$$\frac{1}{d+1} \leq NF(G) \leq 1$$

This result is tight for a graph of max. degree $d = n - 1$.

Lemma Bertrand, Hudry, 2005

Let G be an identifiable graph having no isolated vertices. Let x be a vertex of G. There exists a non forced vertex y in $N[x]$.

Proposition

Let $NF(G)$ be the proportion of non forced vertices of G. Then

$$\frac{1}{d+1} \leq NF(G) \leq 1$$

This result is tight for a graph of max. degree $d = n - 1$.

Lemma Bertrand, Hudry, 2005

Let G be an identifiable graph having no isolated vertices. Let x be a vertex of G. There exists a non forced vertex y in $N[x]$.

Corollary

The set S of non-forced vertices forms a dominating set. Hence $|S| \geq \frac{n}{d+1}$.

clique number of G: max. size of a complete subgraph in G

Proposition

Let G be a graph of clique number at most k. There exists a function c such that:

$$\frac{1}{c(k)} \leq NF(G) \leq 1$$

Bounding the number of forced vertices

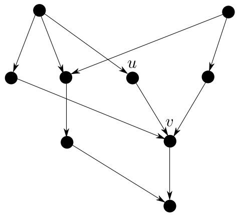
clique number of G: max. size of a complete subgraph in G

Proposition

Let G be a graph of clique number at most k. There exists a function c such that:

$$\frac{1}{c(k)} \leq NF(G) \leq 1$$

- Define graph $\overrightarrow{H}(G)$
- Max. degree of $\overrightarrow{H}(G)$: $2k-3$
- Longest directed chain of $\overrightarrow{H}(G)$: $k-1$
- Each component has a non-forced vertex
- $\Rightarrow c(k) \leq \sum_{i=0}^{k-2}(2k-3)^i$



$u \to v \Leftrightarrow N[v] = N[u] \cup \{x\}$

Corollaries

Theorem (F., Perarnau, 2011+)

There exists an integer d_0 such that for each identifiable graph G on n vertices having maximum degree $d \geq d_0$ and no isolated vertices,

$$\gamma^{\text{ID}}(G) \leq n - \frac{n \cdot NF(G)^2}{85d}$$

Corollary

- In general, $NF(G) \geq \frac{1}{d+1}$ and $\gamma^{\text{ID}}(G) \leq n - \frac{n}{\Theta(d^3)}$
- If G is d-regular, $NF(G) = 1$ and $\gamma^{\text{ID}}(G) \leq n - \frac{n}{85d}$.
- If G has clique number bounded by k, $NF(G) \geq \frac{1}{c(k)}$ and $\gamma^{\text{ID}}(G) \leq n - \frac{n}{\Theta(d)}$.

Let G be a d-regular graph.

$$\gamma^{\text{ID}}(G) \geq \frac{2n}{d+2} \qquad \text{Karpovsky et al. (1998)}$$

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{d} + c \qquad \text{Conjecture (2009)}$$

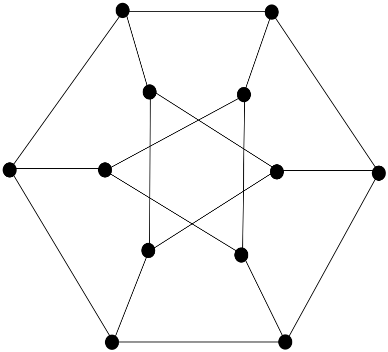
Let G be a d-regular graph.

$$\gamma^{\text{ID}}(G) \geq \frac{2n}{d+2}$$ Karpovsky et al (1998)

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{\Theta(d)}$$ F., Perarnau (2011+)

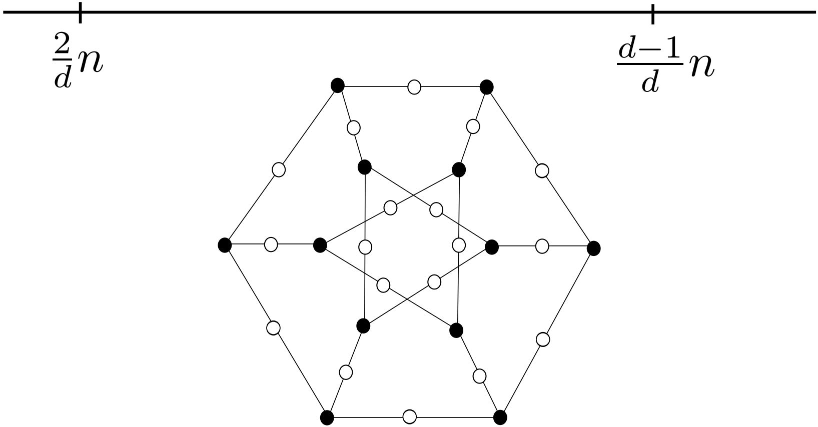
Let G be a d-regular graph.

$\frac{2}{d}n$ — $\frac{d-1}{d}n$

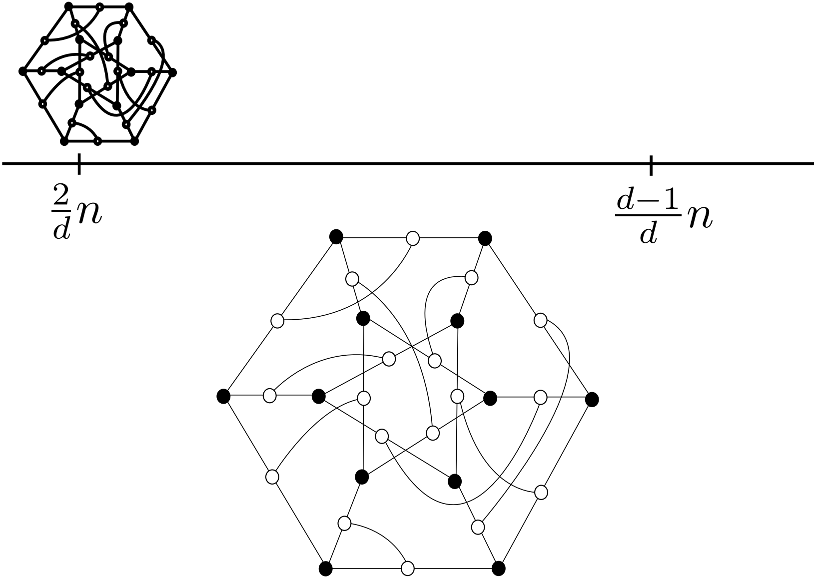


Where are most of the d-regular graphs?

Let G be a d-regular graph.

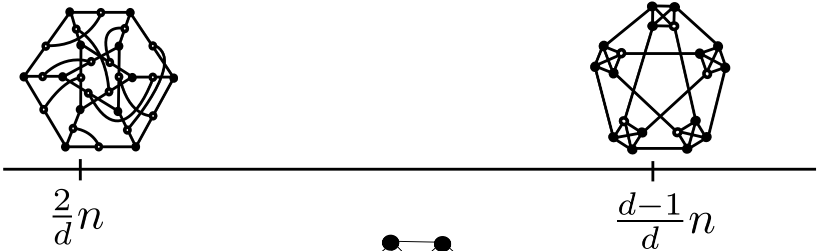


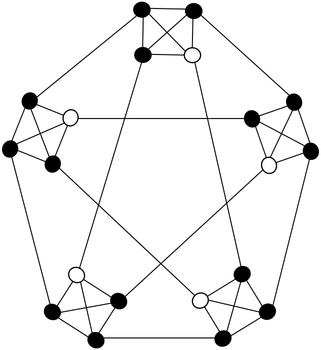
Let G be a d-regular graph.



Where are most of the d-regular graphs?

Let G be a d-regular graph.





Where are most of the d-regular graphs?

Let G be a d-regular graph.

Answer : next week !