Identifying codes in regular graphs

(a probabilistic approach)

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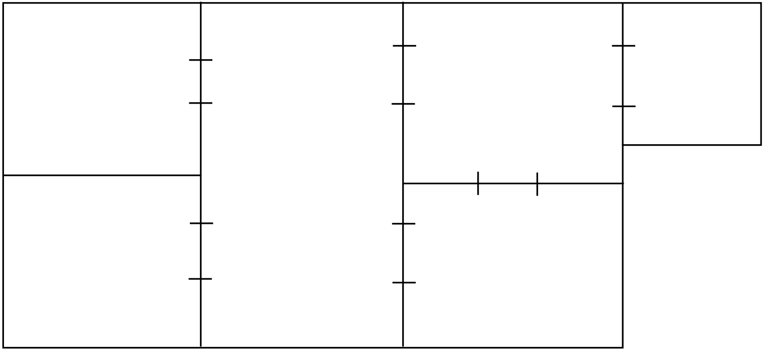
joint work with **Guillem Perarnau** (UPC, Barcelona, Spain)

ANR IDEA

Locating a burglar in a museum



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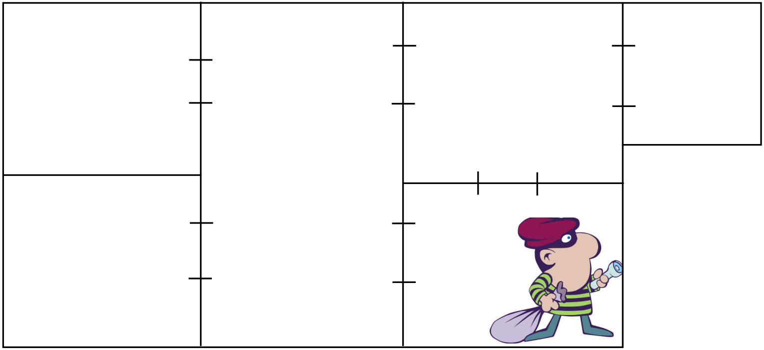
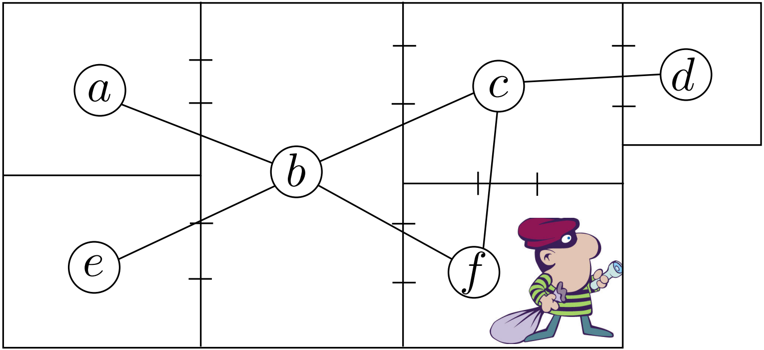
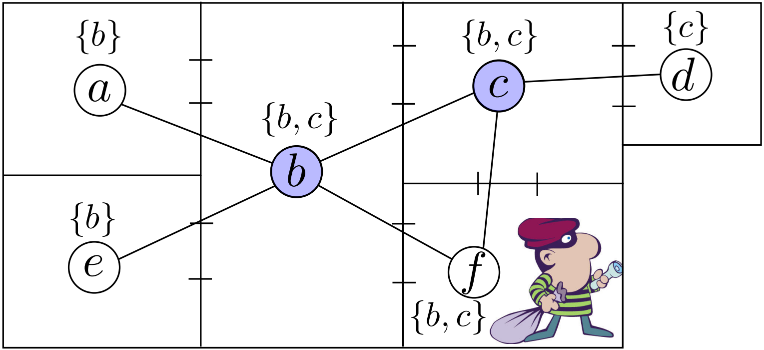




Graph $G = (V, E)$. V: vertices (rooms), $E \subseteq V \times V$: edges (doors)



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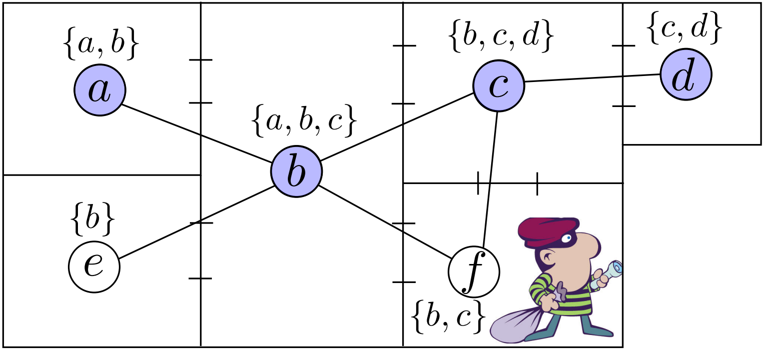

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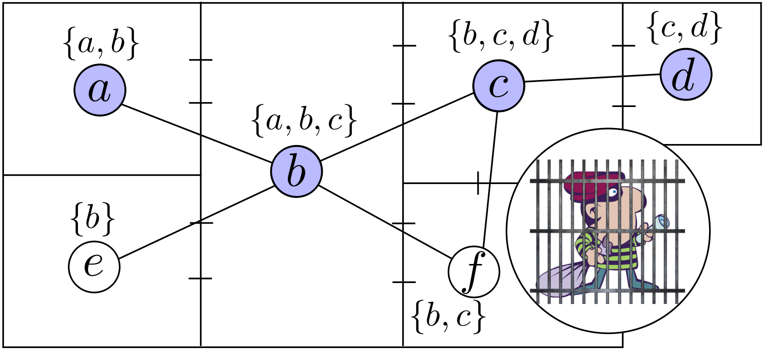

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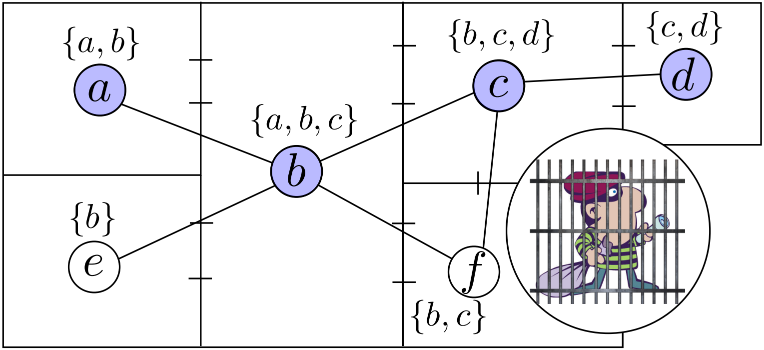

How many **detectors** do we need?

Let $N[u]$ be the set of vertices v s.t. $d(u, v) \leq 1$

Definition - Identifying code of G (Karpovsky, Chakrabarty, Levitin, 1998)

Subset C of V such that:

- C is a **dominating set** in G: $\forall u \in V$, $N[u] \cap C \neq \emptyset$, and
- C is a **separating code** in G: $\forall u \neq v$ of V, $N[u] \cap C \neq N[v] \cap C$

Identifying codes: definition

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Notation - Identifying code number

$\gamma^{\text{ID}}(G)$: minimum cardinality of an identifying code of G

Identifiable graphs

Let $N[u]$ be the set of vertices v s.t. $d(u, v) \leq 1$

Remark

Not all graphs have an identifying code!

Twins = pair u, v such that $N[u] = N[v]$.

A graph is **identifiable** iff it is **twin-free** (i.e. it has no twins).

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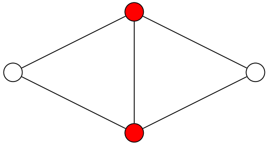

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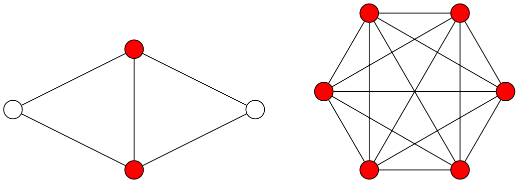
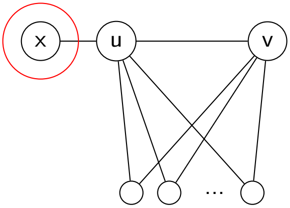


u, v such that $N[v]\Delta N[u] = \{x\}$

Then $x \in C$, forced by uv.



Notation

Let $NF(G)$ be the proportion of **non forced vertices** of G

$$NF(G) = \frac{\#\text{non-forced vertices in G}}{\#\text{vertices in G}}$$

Note: if G regular, $NF(G) = 1$.

Previous results (1/2)

Theorem (Karpovsky, Chakrabarty, Levitin, 1998 + Gravier, Moncel, 2007)

Let G be an identifiable graph with at least one edge, then

$$\lceil \log_2(n+1) \rceil \leq \gamma^{\text{ID}}(G) \leq n-1$$

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Conjecture (F., Klasing, Kosowski, Raspaud, 2009+)

Let G be a connected nontrivial identifiable graph of max. degree d. Then

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{d} + O(1)$$

True for $d = 2$ and $d = n-1$.

Bounds depending on the max. degree

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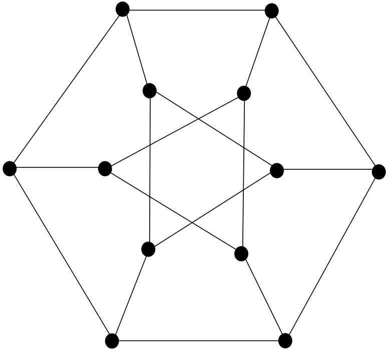

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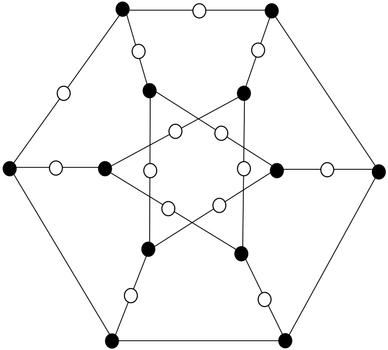

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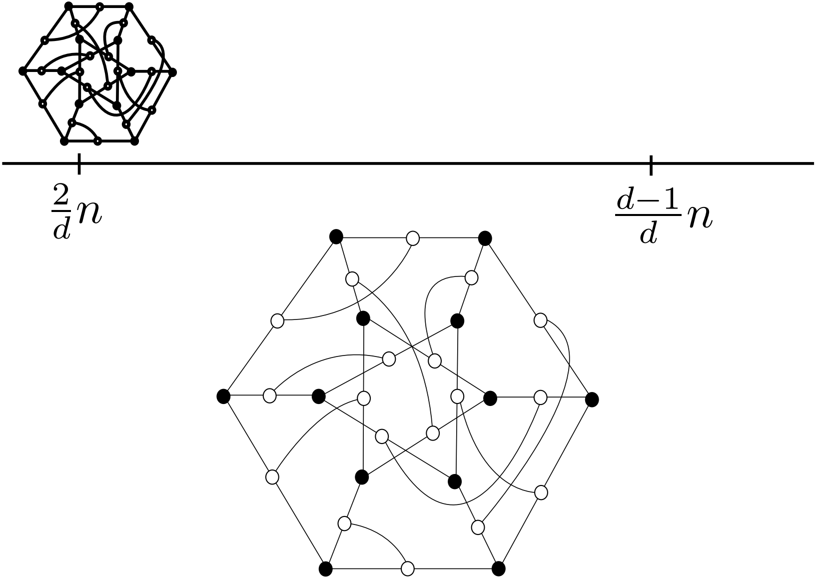

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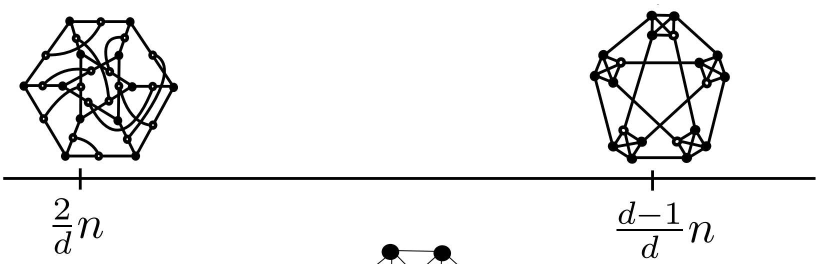

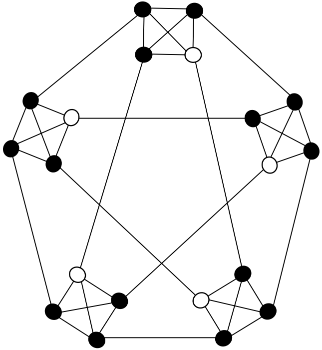

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Theorem (F., Guerrini, Kovse, Naserasr, Parreau, Valicov, 2011)

Let G be a connected identifiable graph of maximum degree d. Then

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{\Theta(d^5)}$$

If G is d-regular, $\gamma^{\text{ID}}(G) \leq n - \frac{n}{\Theta(d^3)}$

Upper bounds for $\gamma^{\text{ID}}(G)$

Notation

Let $NF(G)$ be the proportion of **non** forced vertices of G

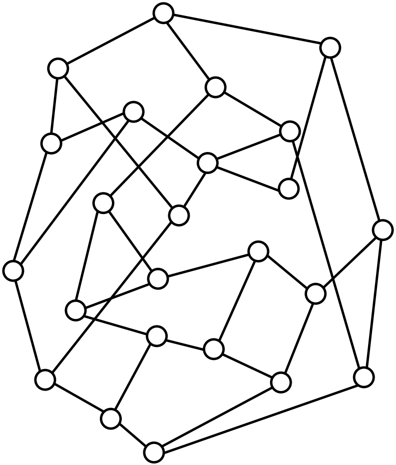
$$NF(G) = \frac{\#\text{non-forced vertices in G}}{\#\text{vertices in G}}$$

Theorem (F., Perarnau, 2011+)

There exists an integer d_0 such that for each identifiable graph G on n vertices having maximum degree $d \geq d_0$ and no isolated vertices,

$$\gamma^{\text{ID}}(G) \leq n - \frac{n \cdot NF(G)^2}{85d}$$

Proof - select a set at random...



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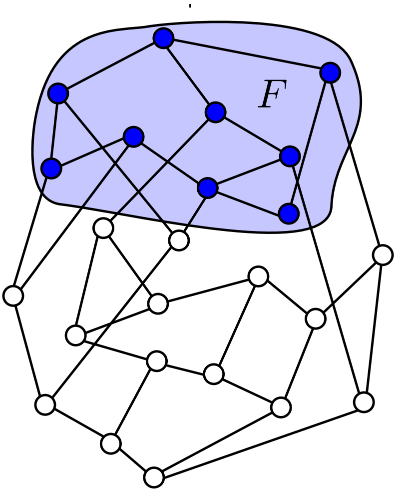

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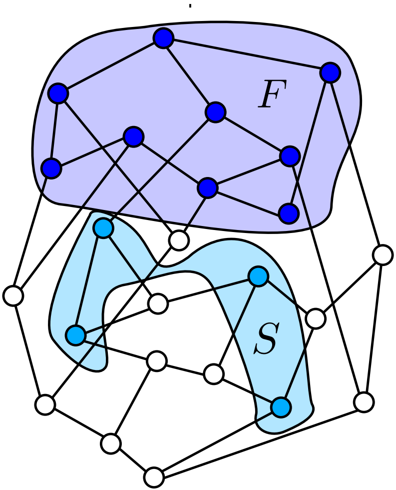
- Select a random set S from $V' = V \setminus F$: each vertex $v \in S$ with prob. p.



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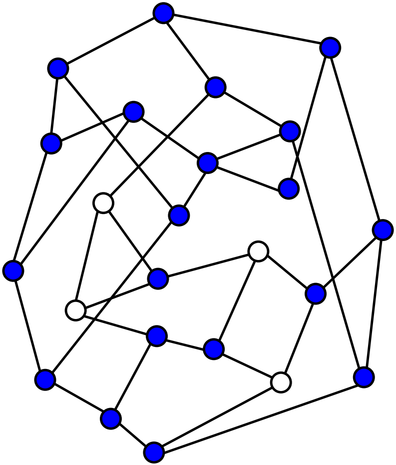

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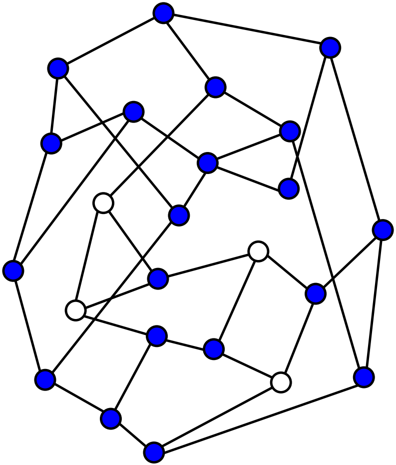


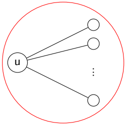
Proof - Using the Lovász Local Lemma

$\mathcal{E} = \{E_1, \ldots, E_M\}$: set of **"bad" events**, dependencies are "rare".

Then: with non-zero probability **none of the bad events occur**.

Moreover, this probability can be lower-bounded.

Set the bad events...



$p^{d+1} \leq Pr(A_u) \leq p^2$

Event A_u

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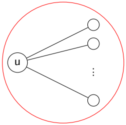

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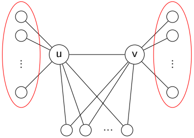

Event $B_{u,v}$

$$p^{2d-2} \leq Pr(B_{u,v}) \leq p^2$$

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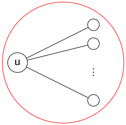

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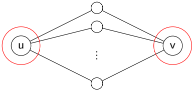

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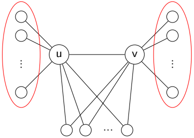

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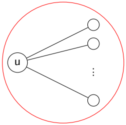

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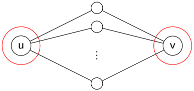

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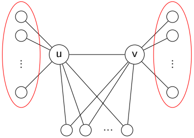

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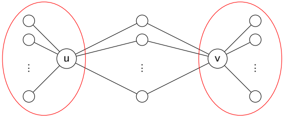

Event $D_{u,v}$

$p^{2d} \leq Pr(D_{u,v}) \leq p^4$

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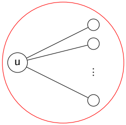

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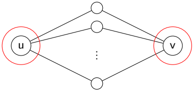

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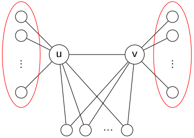

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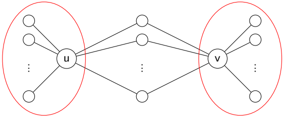

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Taking $p = \frac{1}{kd} \implies$ **LLL can be applied**

Proof - the set can be small...

By the LLL we know that

There exists some set S with $\mathbb{E}(|S|) = \frac{n \cdot NF(G)}{k \cdot d}$ such that no bad event occurs

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But by the LLL we know more:

$$\Pr\left(\bigcap_{i=1}^{m} \overline{E_i}\right) > \exp\left\{-\frac{9}{k^2 d} n\right\}$$

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The probability to have a **good** set S is at least $\exp\left\{-\frac{9}{k^2 d} n\right\}$

Proof (regular case) - concentration inequality

Theorem (Chernoff bound)

Let $X_1, \dots, X_m$ a set of i.i.d random variables s.t. $Pr(X_i = 1) = p$ and $Pr(X_i = 0) = 1 - p$ and $X = \sum X_i$. Then

$$\Pr(\mathbb{E}(X) - X > \alpha) \leq \exp\left\{-\frac{\alpha^2}{2mp}\right\}$$

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For each $v_i \in V \setminus F$ define the random variable:

$$X_i = \begin{cases} 1 & \text{if } v_i \in C \\ 0 & \text{otherwise} \end{cases}$$

Then, we set $\alpha = \frac{n \cdot NF(G)}{cd}$. Using $mp = \frac{n \cdot NF(G)}{kd}$:

$$\Pr\left(\mathbb{E}(X) - X > \frac{n \cdot NF(G)}{cd}\right) \leq \exp\left\{\frac{kNF(G)}{2c^2 d} n\right\}$$

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$$\Pr\left(\mathbb{E}(X) - X > \frac{n \cdot NF(G)}{cd}\right) \leq \exp\left\{\frac{kNF(G)}{2c^2d}n\right\}$$

Probability that S is **too small**: at most $\exp\left\{-\frac{kNF(G)}{2c^2d}n\right\}$

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$$\Pr(S \text{ good}) - \Pr(S \text{ too small}) > 0$$

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$$|\mathcal{C}| = |V \setminus S| \leq n - \frac{n \cdot NF(G)^2}{85d}$$

Theorem (F., Perarnau, 2011+)

There exists an integer d_0 such that for each identifiable graph G on n vertices having maximum degree $d \geq d_0$ and no isolated vertices,

$$\gamma^{\text{ID}}(G) \leq n - \frac{n \cdot NF(G)^2}{85d}$$

Proposition

Let $NF(G)$ be the proportion of **non forced vertices** of G. Then

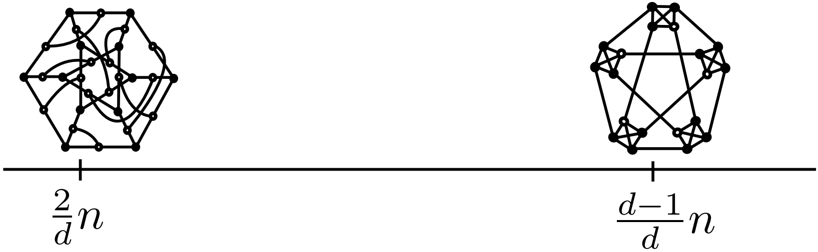
$$\frac{1}{d+1} \leq NF(G) \leq 1$$

Corollary

- In general, $NF(G) \geq \frac{1}{d+1}$ and $\gamma^{\text{ID}}(G) \leq n - \frac{n}{\Theta(d^3)}$
- If G is d-regular, $NF(G) = 1$ and $\gamma^{\text{ID}}(G) \leq n - \frac{n}{85d}$.

Where are most of the d-regular graphs?

Let G be a d-regular graph.



$\gamma^{\text{ID}}(G) \geq \frac{2n}{d+2}$ Karpovsky et al. (1998)

$\gamma^{\text{ID}}(G) \leq n - \frac{n}{d} + O(1)$ Conjecture (2009)

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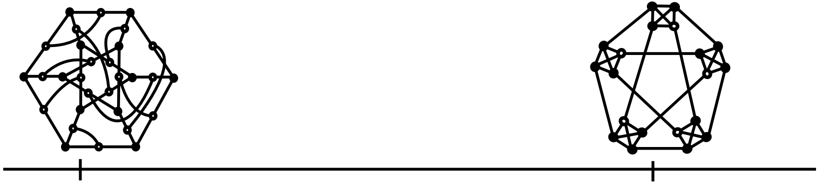

$\frac{2}{d}n$ $\qquad\qquad\qquad\qquad$ $\frac{d-1}{d}n$

$$\gamma^{\text{ID}}(G) \geq \frac{2n}{d+2} \qquad \text{Karpovsky et al. (1998)}$$

$$\gamma^{\text{ID}}(G) \leq n - \frac{n}{\Theta(d)} \qquad \text{F., Perarnau (2011+)}$$

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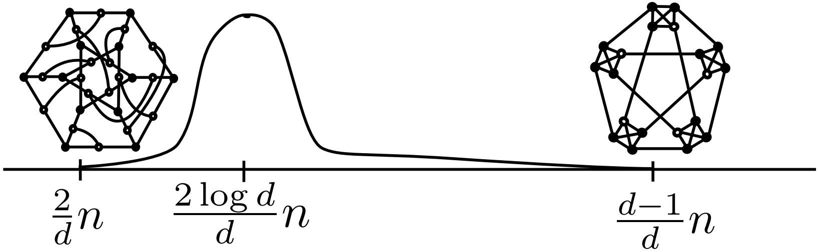

Theorem (F., Perarnau, 2011+)

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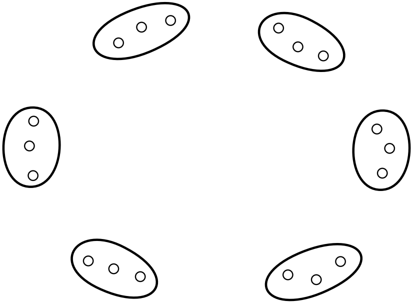
$$(1+o_d(1))\frac{\log d}{d}n \leq \gamma^{\text{ID}}(G) \leq (1+o_d(1))\frac{2\log d}{d}n$$

The pairing model (a.k.a. configuration model) - Bollobás, 1980

Probability space $\mathcal{G}^*_{n,d}$ of d-regular **multigraphs** on n vertices.



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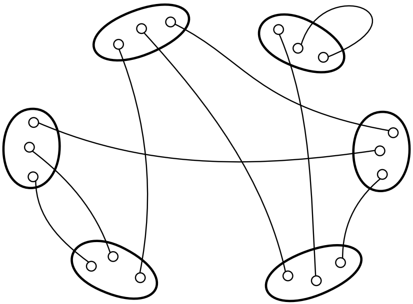

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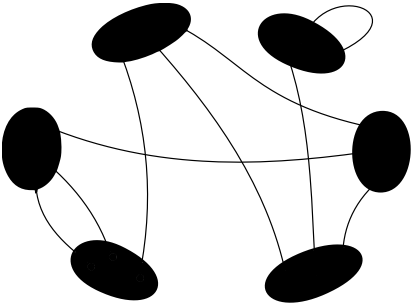

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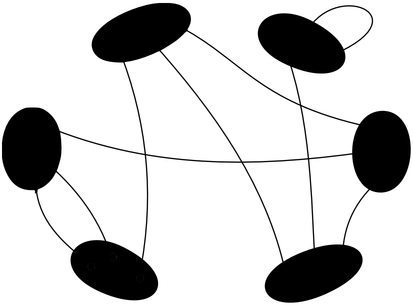

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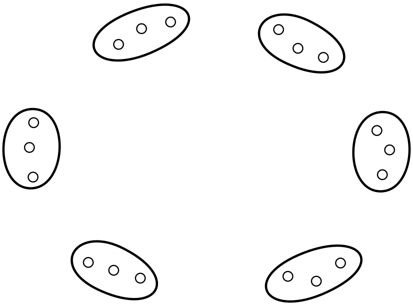

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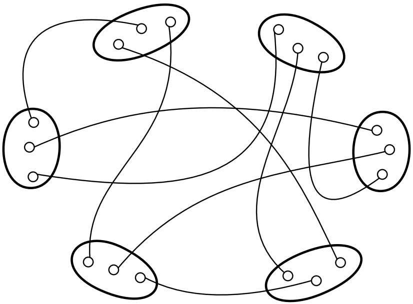

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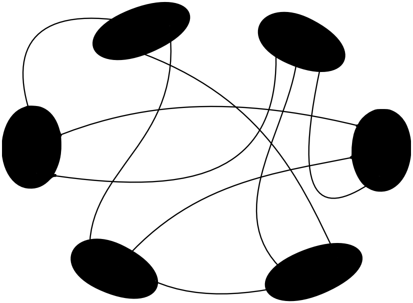

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Let $G \in \mathcal{G}^*_{n,d}$. Then $Pr(G \text{ is simple}) \longrightarrow e^{\frac{1-d^2}{4}} > 0$
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Graphs with girth at least 5

Proposition (F., Perarnau, 2011+)

Let G be a d-regular graph with girth at least 5. Then

$$\gamma^{\text{ID}}(G) \leq (1 + o_d(1))\frac{2\log d}{d}n$$

2-dominating is **“almost sufficient”** to identify.

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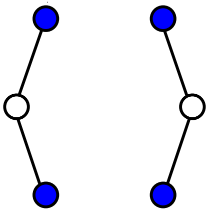

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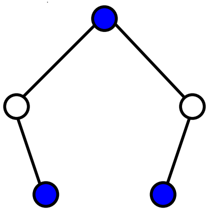

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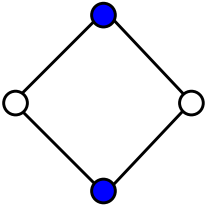

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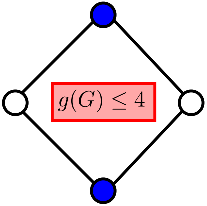
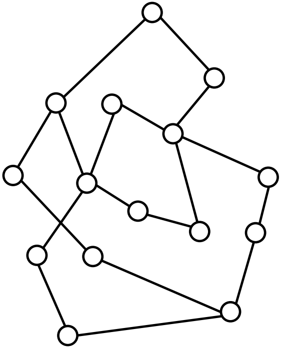


$g(G) \geq 5$ makes identifying easier.

Sketch of the proof: construct 2-dominating set D

- $S \subseteq V$ at random, each element with probability p.



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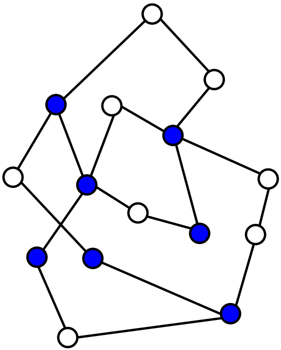

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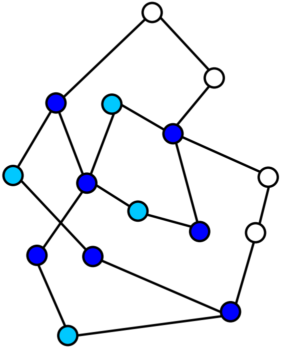
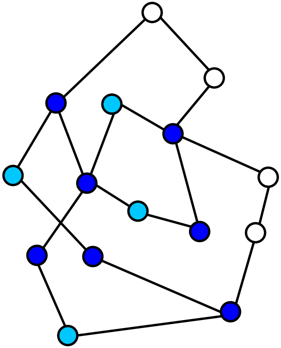


- $S \subseteq V$ at random, each element with probability p.
- $$X_v = \begin{cases} 0 & \text{if } |N[v] \cap S| \geq 2 \\ 1 & \text{otherwise} \end{cases}$$

  $$Pr(X_v = 1) = (1-p)^{d+1} + (d+1)p(1-p)^d$$



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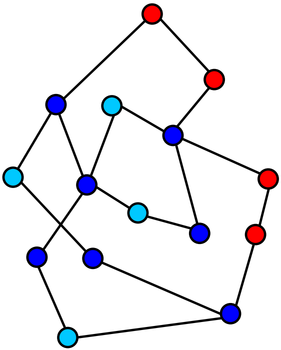

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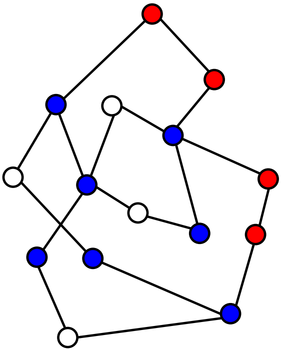
$$\mathbb{E}(|D|) = \mathbb{E}(|S|) + X(S) \leq \frac{2 \log d}{d} n$$



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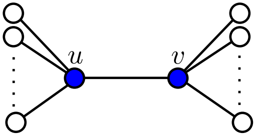

PROBLEM!

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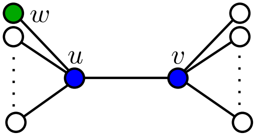
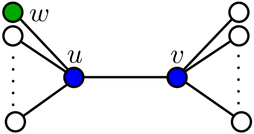


$SOLVED$!



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$$Y_{uv} = \begin{cases} 1 & \text{if } \end{cases}$$ 

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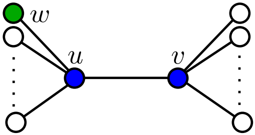

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Back to random regular graphs - upper bound

Theorem (F., Perarnau, 2011+)

Let G be a random d-regular graph. Then a.a.s.

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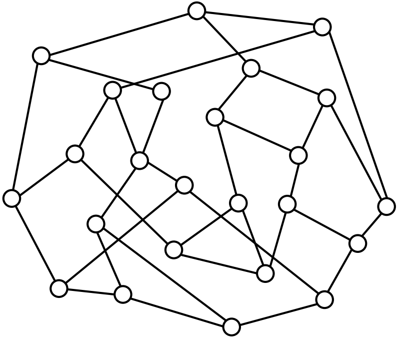
Let G be a d-regular graph of order n, taken **u.a.r.**: $G \in \mathcal{G}(n, d)$



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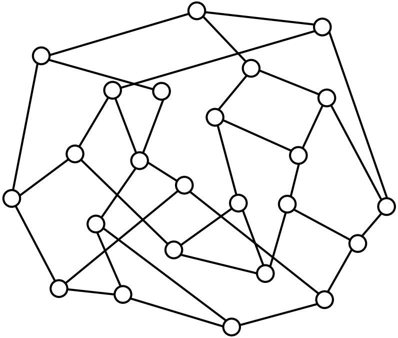

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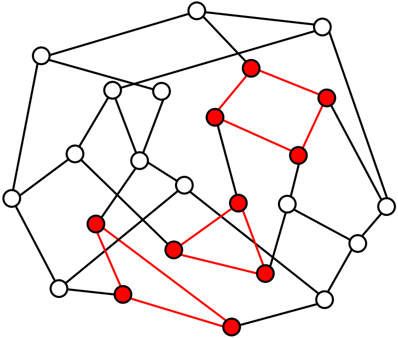

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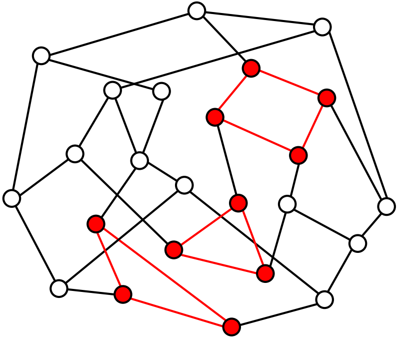

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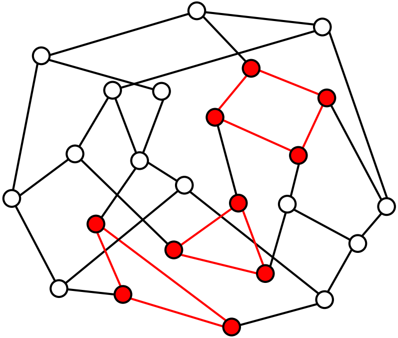

$$\gamma^{\text{ID}}(G) \leq |\mathcal{C}| = (1 + o_d(1))\frac{2\log d}{d}n \textbf{ a.a.s.}$$

Thank you!