Bounding K_4-minor-free graphs in the homomorphism order

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November 24th, 2012

BGW 2012

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Definition - Graph homomorphism from G to H

Mapping from $V(G)$ to $V(H)$ which **preserves adjacency**.
If it exists, we note $G \to H$.

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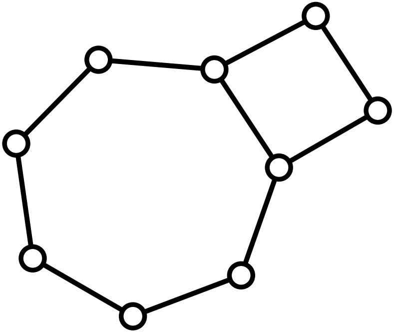

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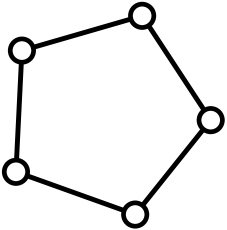

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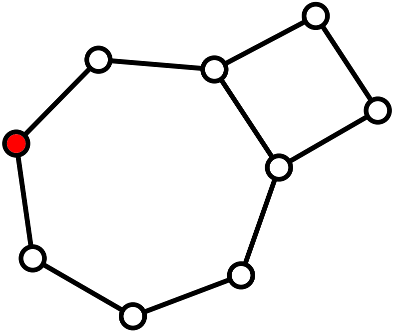

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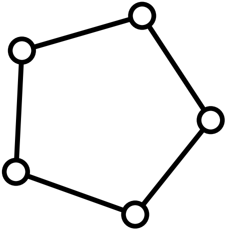

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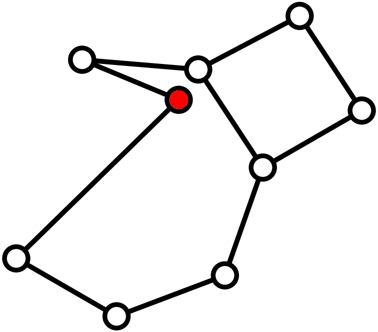

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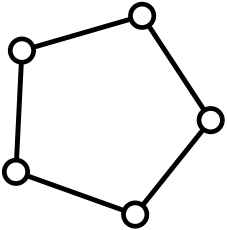

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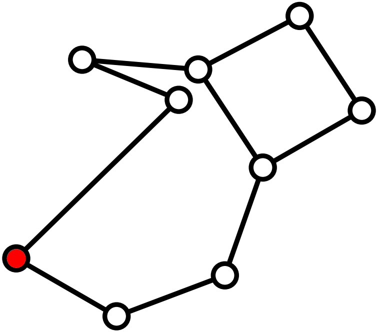

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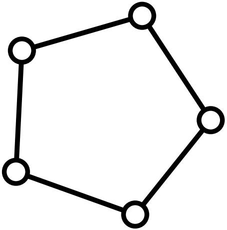

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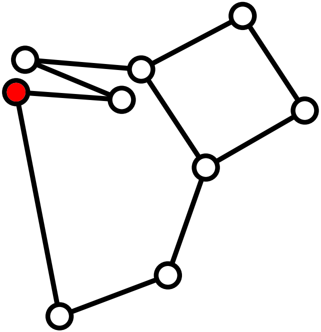

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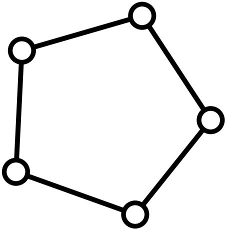

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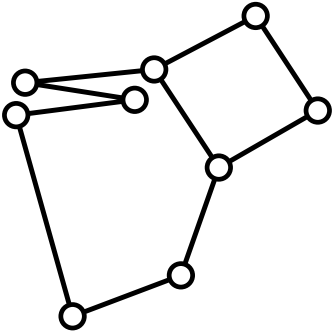

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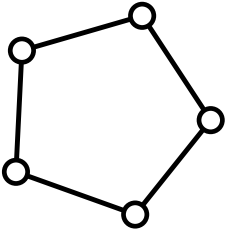

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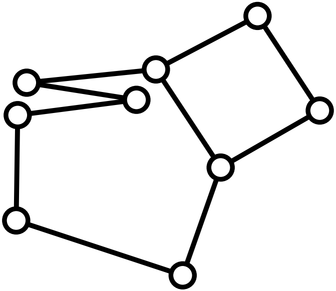

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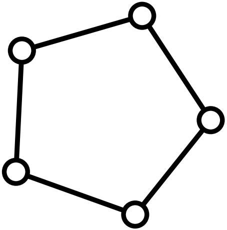

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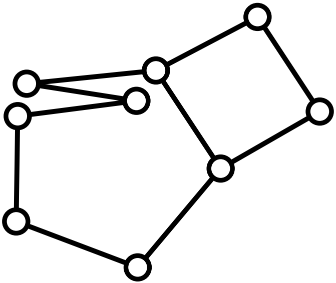

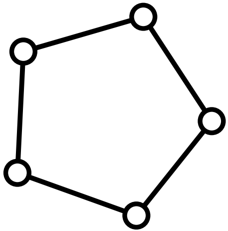

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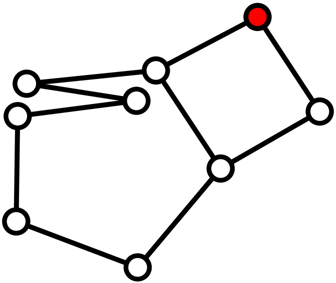

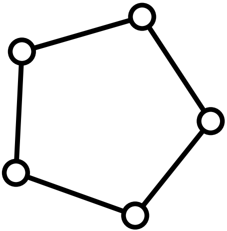

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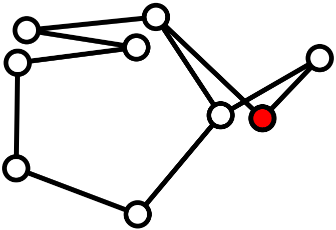

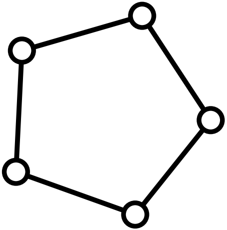

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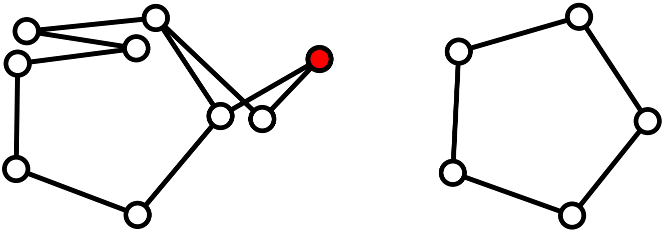

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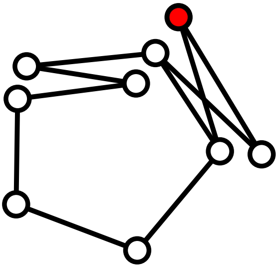

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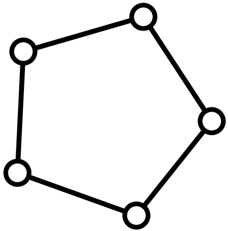

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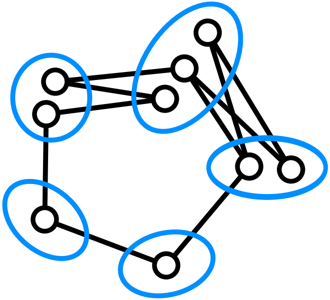

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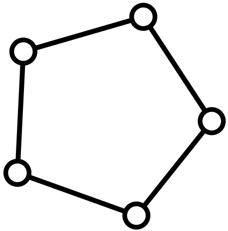

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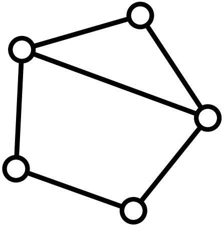

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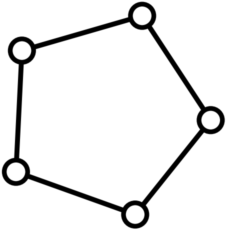

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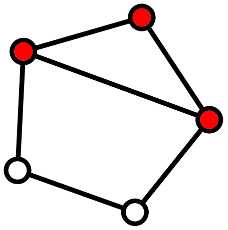

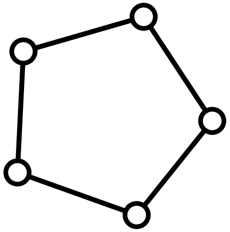

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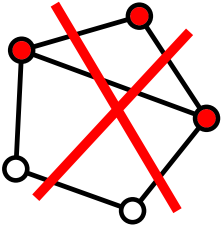

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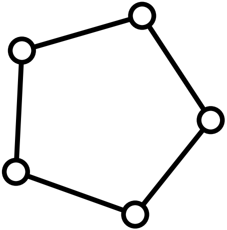
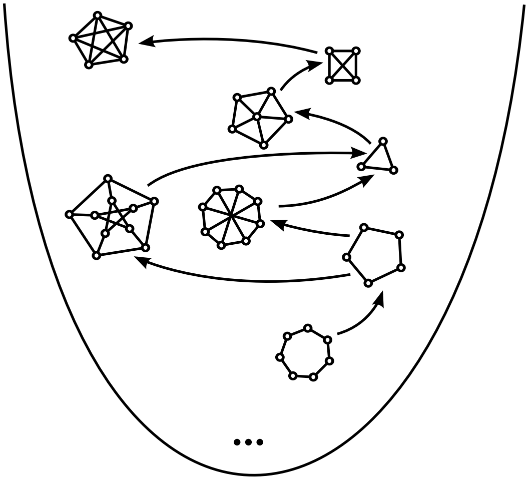


The homomorphism order

Definition - Homomorphism quasi-order

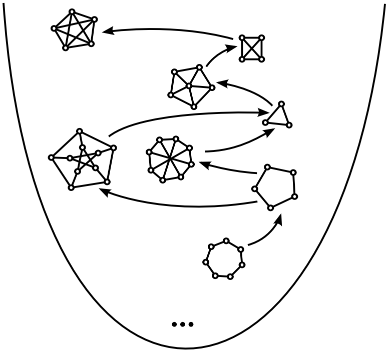
Defined by $G \preceq H$ iff $G \to H$ (if restricted to cores: partial order).



Bounds in the homomorphism order

Definition - Bound in the order

$B_{\mathcal{C}}$ is a **bound** for some graph class $\mathcal{C}$ if for each $G \in \mathcal{C}$, $G \to B_{\mathcal{C}}$.



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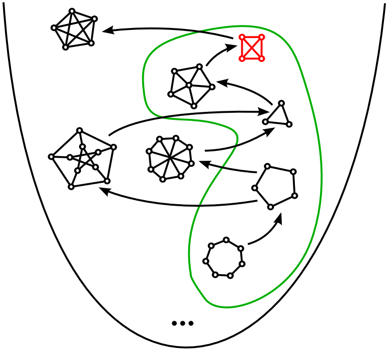

K_4 is a bound for all planar graphs (4CT)

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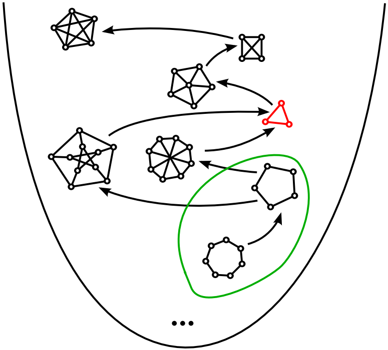

K_3 is a bound for all planar triangle-free graphs (Grötzsch's theorem)

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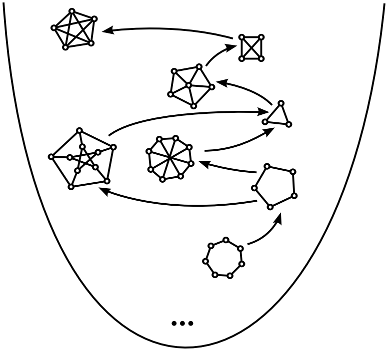

Question

Given graph class $\mathcal{C}$, is there a bound for $\mathcal{C}$ having **specific properties**?

Nešetřil-Ossona de Mendez theorem

Definition

$\mathcal{F}$: finite set of graphs.
Forb($\mathcal{F}$): set of graphs G s.t. for any $F \in \mathcal{F}$, $F \not\to G$.

Examples:

- *Forb*(K_ℓ): graphs with **clique number** at most $\ell - 1$
- *Forb*(C_{2k-1}): graphs of **odd girth** at least $2k + 1$
  (odd girth: length of a smallest odd cycle)

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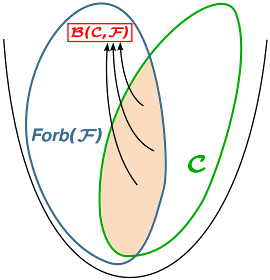
Theorem (Nešetřil and Ossona de Mendez, 2008)

For any **minor-closed** class $\mathcal{C}$ of graphs, $\mathcal{C} \cap Forb(\mathcal{F})$ is **bounded** by a finite graph $\mathcal{B}(\mathcal{C},\mathcal{F})$ from *Forb*($\mathcal{F}$).



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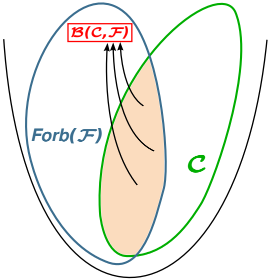

Example: $\mathcal{C}$={planar graphs}
$\mathcal{F} = \{C_{2k-1}\}$

$\longrightarrow$ all planar graphs of odd girth at least $2k+1$ map to some graph $B_{n,k}$ of odd girth $2k+1$.

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Question

When a bound exists, which is a bound of **smallest order**?

Example: Hadwiger's conjecture: smallest B_n is K_{n-1}.

Naserasr's conjecture and projective cubes

Conjecture (Naserasr, 2007)

The class of **planar** graphs of odd girth at least $2k + 1$ is bounded by the **projective cube** $PC(2k)$, and this bound is **optimal**.

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Obtained from hypercube $H(d)$ by **adding edges** between all **antipodal pairs**.

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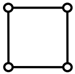
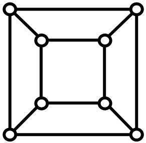


$H(2)$



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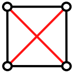
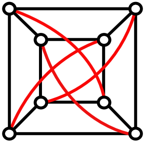


$PC(2) = K_4$



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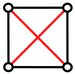

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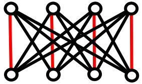

$PC(3) = K_{4,4}$

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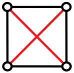

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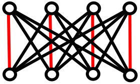

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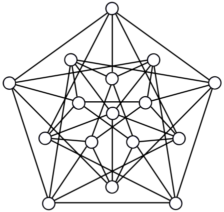

$PC(4)$: Clebsch graph

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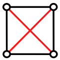
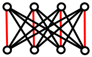
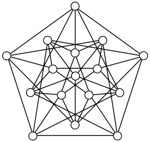


$PC(2) = K_4$



$PC(3) = K_{4,4}$



$PC(4)$: Clebsch graph

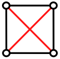
Remark

$PC(d)$ is **distance-transitive**: for any two pairs $\{x, y\}$, $\{u, v\}$ with $d(x, y) = d(u, v)$, there is an automorphism with $x \to u$ and $y \to v$

Projective cubes

Definition - Projective cube of dimension d, $PC(d)$

Obtained from hypercube $H(d)$ by **adding edges** between all **antipodal pairs**.



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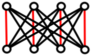

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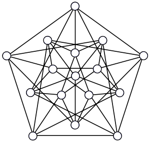

$PC(4)$: Clebsch graph

Remark

$d = 2k + 1$ odd: $PC(2k + 1)$ bipartite
$d = 2k$ even: $PC(2k)$ has odd girth $2k + 1$

Naserasr's conjecture

Conjecture (Naserasr, 2007)

The class of **planar** graphs of odd girth at least $2k+1$ is bounded by the **projective cube** $PC(2k)$, and this bound is **optimal**.

Naserasr's conjecture

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The class of **planar** graphs of odd girth at least $2k + 1$ is bounded by the **projective cube** $PC(2k)$, and this bound is **optimal**.

Conjecture (Seymour, 1981)

Every planar r-graph is r-edge-colourable.

(r-graph: r-regular multigraph without odd ($< r$)-cut)

Theorem (Naserasr, 2007)

The class of planar graphs of odd girth at least $2k + 1$ is bounded by $PC(2k)$ if and only if every planar $(2k + 1)$-graph is $(2k + 1)$-edge-colourable.

Naserasr's conjecture

Conjecture (Naserasr, 2007)

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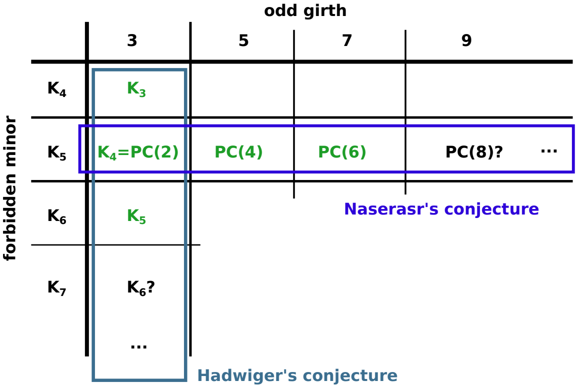

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forbidden minor \ odd girth	3	5	7	9	
K_4	K_3	???	???	???	...
K_5	K_4=PC(2)	PC(4)	PC(6)	PC(8)?	...
K_6	K_5				
K_7	K_6?				
	...				

Naserasr's conjecture

Hadwiger's conjecture

K_4-minor-free graphs

Question

What is a (good) bound of odd girth $2k+1$ for K_4-minor-free graphs of odd girth at least $2k+1$?

K_4-minor-free graphs

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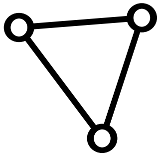
What is a (good) bound of odd girth $2k+1$ for K_4-minor-free graphs of odd girth at least $2k+1$?

Proposition

A graph is K_4-minor free if and only if it is a partial 2-tree.



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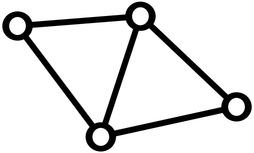

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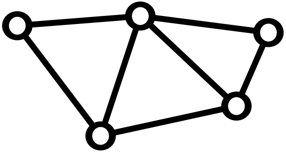

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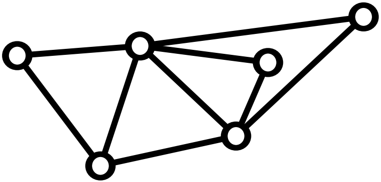

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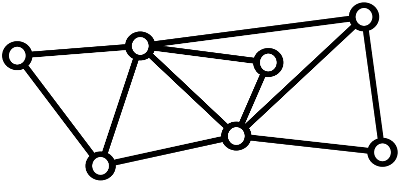

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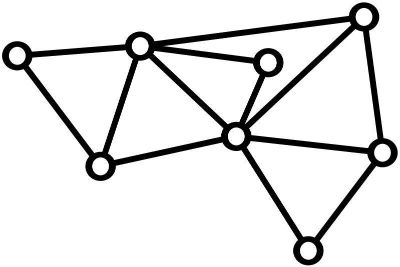

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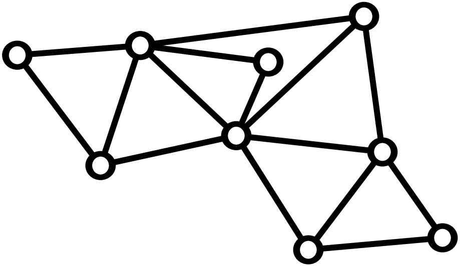

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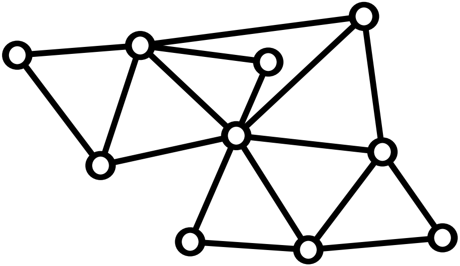

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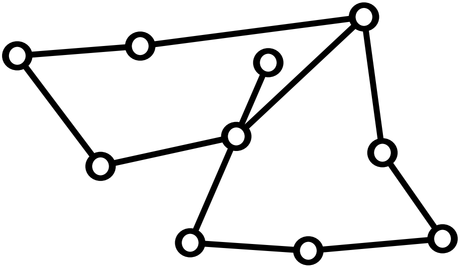


K_4-minor-free graphs and projective cubes

Conjecture (Naserasr, 2007)

The class of **planar** graphs of odd girth at least $2k+1$ is bounded by the **projective cube** $PC(2k)$, and this bound is **optimal**.

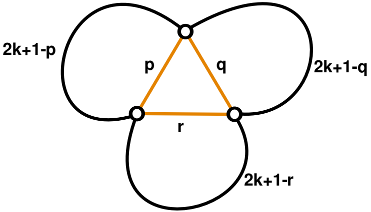
Theorem

$PC(2k)$ is a bound for K_4-minor-free graphs of odd girth at least $2k+1$.

$PC(2k)$ is a bound for K_4-minor-free graphs of odd girth at least $2k+1$

Proof idea: use partial 2-tree structure + good properties of $PC(2k)$.

1. Define "allowed distance triples" $\{p, q, r\}$ $(1 \leq p, q, r \leq 2k)$.



$\{p, q, r\}$ **allowed triple** if it does not create a short odd cycle.

$PC(2k)$ is a bound for K_4-minor-free graphs of odd girth at least $2k+1$

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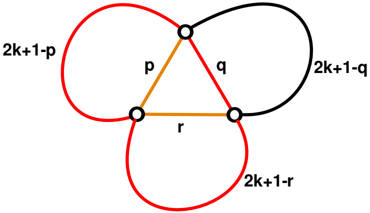

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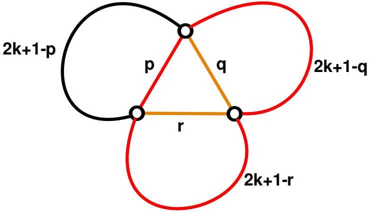

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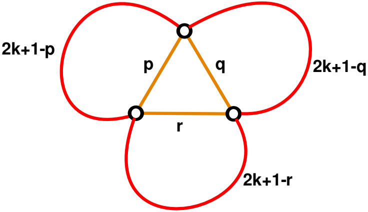

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2. *Lemma:* for each pair u, v of vertices of $PC(2k)$ at distance d, all allowed triples $\{d, x, y\}$ are realized on u, v.

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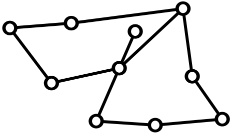
3. Let G be a minimum counterexample.



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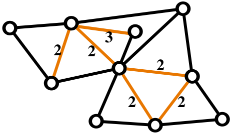
3. Let G be a minimum counterexample.
Make it K_4-minor-free edge-maximal with distance labels



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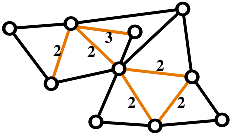

4. *Lemma (Nešetřil-Nigussie, 2007):* all "triangles" form allowed triples.

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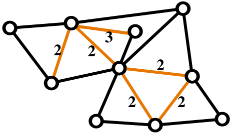

4. *Lemma (Nešetřil-Nigussie, 2007):* all "triangles" form allowed triples.

5. Use the 2-tree structure in a greedy way to map it: contradiction. □

K_4-minor-free graphs, corollary

Theorem

$PC(2k)$ is a bound for K_4-minor-free graphs of odd girth at least $2k+1$.

odd girth / forbidden minor

forbidden minor \ odd girth	3	5	7	9	
K_4	K_3	PC(4)	PC(6)	PC(8)	...
K_5	K_4=PC(2)	PC(4)	PC(6)	PC(8)?	...
K_6	K_5				
K_7	K_6?				
	...				

Naserasr's conjecture (row K_5)

Hadwiger's conjecture (column 3)

K_4-minor-free graphs, corollary

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$PC(2k)$ is a bound for K_4-minor-free graphs of odd girth at least $2k+1$.

odd girth / forbidden minor

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K_6	K_5				
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	...				

Naserasr's conjecture

Hadwiger's conjecture

Corollary

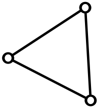
Every K_4-minor-free $(2k+1)$-graph is $(2k+1)$-edge-colourable.

(result already known)

K_4-minor-free graphs, finding better bounds

Theorem

- K_3 is the smallest bound for K_4-minor-free graphs (well-known).

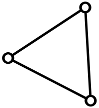


odd girth 3: K_3

K_4-minor-free graphs, finding better bounds

Theorem

- K_3 is the smallest bound for K_4-minor-free graphs (well-known).
- The Wagner graph is the smallest bound of odd girth 5 for K_4-minor-free graphs of odd girth at least 5.



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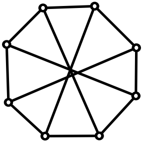

odd girth 5: Wagner graph

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- K_3 is the smallest bound for K_4-minor-free graphs (well-known).
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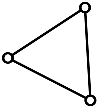
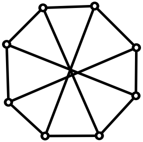
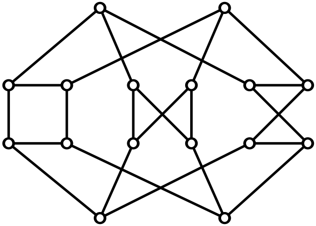
- G_{16} is a bound of odd girth 7 for K_4-minor-free graphs of odd girth at least 7.



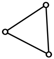
odd girth 3: K_3



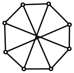
odd girth 5: Wagner graph



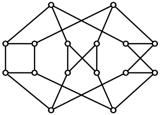
odd girth 7: G_{16}



odd girth 3:
K_3



odd girth 5:
Wagner graph



odd girth 7:
G_{16}

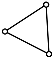
odd girth 9:
???

forbidden minor \ odd girth	3	5	7	9	
K_4	K_3	Wagner	G_{16} (smallest?)	???	
K_5	K_4=PC(2)	PC(4)	PC(6)	PC(8)?	...
K_6	K_5				
K_7	K_6?				
	...				

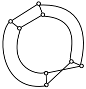
Naserasr's conjecture

Hadwiger's conjecture

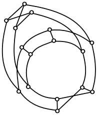
K_4-minor-free graphs, finding better bounds



odd girth 3:
K_3



odd girth 5:
Wagner graph



odd girth 7:
G_{16}

odd girth 9:
???

forbidden minor	odd girth 3	5	7	9	
K_4	K_3	Wagner	G_{16} (smallest?)	???	
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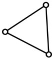
Naserasr's conjecture

Hadwiger's conjecture

K_4-minor-free graphs, finding better bounds



odd girth 3:
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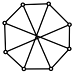
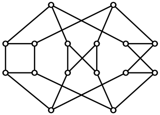


odd girth 5:
Wagner graph



odd girth 7:
G_{16}

odd girth 9:
???

odd girth / **forbidden minor**

	3	5	7	9	
K_4	K_3	Wagner	G_{16} (smallest?)	???	
K_5	$K_4=PC(2)$	$PC(4)$	$PC(6)$	$PC(8)$?	...
K_6	K_5				
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	...				

Naserasr's conjecture

Hadwiger's conjecture

Remark

$K_3 \subseteq K_4 = PC(2)$
Wagner graph $\subseteq PC(4)$
$G_{16} \subseteq PC(6)$